Unusual behaviour of epicyclic frequencies around rapidly rotating compact stars

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ABSTRACT
We report on numerical calculations of orbital and epicyclic frequencies in nearly circular orbits around rotating neutron stars and strange quark stars. The FPS equation of state was used to describe the neutron star structure while the MIT bag model was used to model the equation of state of strange quark stars. The uniformly rotating stellar configurations were computed in full general relativity. We find that the vertical epicyclic frequency is very sensitive to the oblateness of the rotating star. For models of rotating neutron stars of moderate mass, as well as for strange quark star models, the sense of the nodal precession of test particle orbits close to the star changes at a certain stellar rotation rate. These findings may have implications for models of kHz QPOs.

Keywords: epicyclic frequencies – neutron star – strange star – quark star – general relativity – numerical relativity – quasi-periodic oscillations

1 INTRODUCTION

The discovery of kHz Quasi-periodic oscillations (QPOs) is among the most important scientific result of Rossi X-ray Timing Explorer (RXTE). To date, kHz QPOs have been discovered in about 20 neutron star low-mass X-ray binaries (LMXBs), which typically exhibit two high frequency peaks in the power spectra of the X-ray flux. The QPO phenomenon promises to be a probe of the innermost regions of accretion disks around compact objects such as white dwarfs, neutron stars and black holes, although the promise has not yet been fully realized (see van der Klis, 2000, for a review). Most models of kHz QPOs involve orbital and epicyclic frequencies (Kato and Fukue, 1980; Nowak and Wagoner, 1991, 1992; Perez et al., 1997; Stella et al., 1999; Wagoner, 1999; Abramowicz and Kluźniak, 2001; Silbergleit et al., 2001; Kluźniak, 2005; Stuchlík et al., 2012, 2013).

In the case of Newtonian gravity of a spherically symmetric body the three basic frequencies associated with nearly circular motion, i.e. the orbital (Ω_K), radial epicyclic (ω_r), and vertical epicyclic (ω_z) frequencies are equal to each other. In the case of black holes,

the radial epicyclic frequency is lower than the orbital one, both in the Schwarzschild and the Kerr metrics. For prograde orbits of the Kerr black hole, the vertical epicyclic frequency is lower than the orbital frequency, but higher than the radial epicyclic one, while for retrograde orbits the vertical epicyclic is larger than the orbital frequency (Perez et al., 1997).

However, in Newtonian gravity the degeneracy between the three frequencies can be broken by rotation. It has been shown that for extremely oblate bodies in strictly Newtonian gravity the radial epicyclic frequency may even go to zero at a certain distance from the body (and be imaginary closer to it), so that no stable orbits will exist close to a very rapidly rotating fluid configuration (Kluźniak et al., 2001; Zdunik and Gourgoulhon, 2001). In particular, no stable circular orbits exist right outside Maclaurin spheroids of ellipticity $e > 0.834583178$ (Amsterdamski et al., 2002). Further, it has been shown that in Newton's gravity, the ordering of the frequencies around an oblate body such as a Maclaurin spheroid is $\omega_r < \Omega_K < \omega_z$ (Kluźniak and Rosińska, 2013). In summary, the Newtonian effects of oblateness are the opposite of those of frame-dragging in Kerr geometry for prograde orbits: the innermost stable circular orbit is pushed away from the gravitating body and the vertical epicyclic frequency is increased.

For rotating bodies in general relativity (GR) there is a competition between frame-dragging and effects of oblateness (Stergioulas et al., 1999). Effects of oblateness of the gravitating rotating body have also been noted in GR in the context of the "relativistic precession model" of neutron star kHz QPOs, which relies on the differences between the orbital and epicyclic frequencies (Morsink and Stella, 1999). In the case of rapidly rotating strange quark stars Gondek-Rosińska et al. (2014), computed the frequencies for two stellar masses ($M = 1.4\,M_\odot$ and $M = 1.96\,M_\odot$), and showed that the vertical epicyclic frequency and the related nodal precession rate of inclined orbits are very sensitive to the oblateness of the rotating star. In particular, for rotating stellar models of moderate and high-mass strange quark stars, the sense of the nodal precession (given by the sign of $\Omega_K - \omega_z$) changes at a certain rotation rate. We defer a discussion of the potential astrophysical implications of this finding till Section 3.

We report on numerical calculations of orbital and epicyclic frequencies for rotating strange quark stars and neutron stars for a wide range of masses and two rotation rates. We have used the RNS code for our calculations (Stergioulas and Friedman, 1995). In this contribution we are discussing the similarities between the behaviour of epicyclic frequencies in strange stars and in neutron stars modelled with the FPS equation of state.

2   PROPERTIES OF CIRCULAR ORBITS IN THE METRIC OF ROTATING NEUTRON STARS AND QUARK STARS

We have performed all of our numerical calculations of strange quark stars in the framework of the MIT bag model (Farhi and Jaffe, 1984). In this model quark matter is composed of massless up and down quarks, massive strange quarks and electrons. We use the simplest MIT bag model with massless strange quarks, with the equation of state given by $P = a(\rho - \rho_0)c^2$, where P is the pressure, ρ is mass-energy density, and c is the speed of light. We take $a = 1/3$ and $\rho_0 = 4.2785 \times 10^{14}$ g/cm^3.
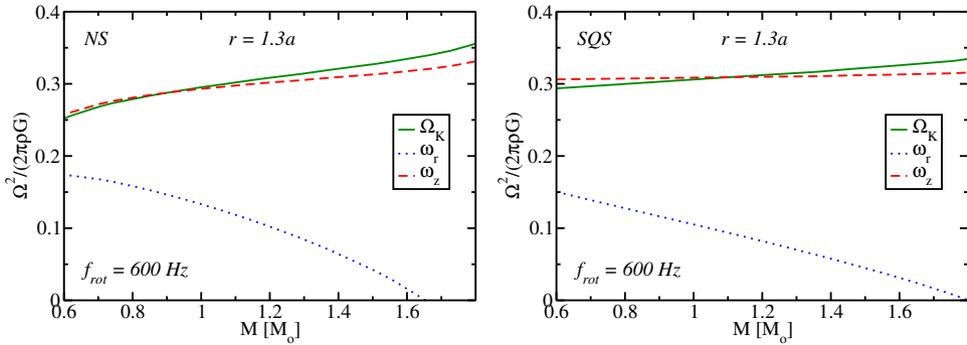
Figure 1. Squares of the orbital and epicyclic frequencies (scaled with $2\pi G\rho$) at $r = 1.3\,a$ versus gravitational mass for uniformly rotating neutron stars (*left panel*), and strange quark stars (*right panel*), rotating at a fixed frequency $f_{\rm rot} = 600\,$Hz. The solid (*green*) line corresponds to orbital frequency, the dashed (*red*) line to vertical epicyclic frequency and the dotted (*blue*) line to radial epicyclic frequency.

For computing neutron star models we have used a modern version of FPS equation of state (see Cook et al., 1994, for a review) proposed by Friedman and Pandharipande (1981).

The orbital and epicyclic frequencies are exhibited for a wide range of masses of strange stars and neutron stars, for two stellar rotation rates, 600 Hz and 900 Hz. In the former case (Fig. 1) we present the frequencies squared for orbits at some distance from the star (at $r = 1.3\,a$), while in the latter (Fig. 2) the same quantities are displayed for orbits grazing the stellar equator ($r = a$).

The effects of oblateness on the epicyclic frequencies in numerical solutions for a neutron star rotating at 400 Hz has been clearly seen in the unusually small difference $\Omega_{\rm K} - \omega_z > 0$ between the orbital frequency and the vertical epicyclic one (Kluźniak et al., 2004). It had been expected that frame-dragging effects dominate those of oblateness (Kluźniak, 1998; Morsink and Stella, 1999). However, we now show for the first time that the vertical epicyclic frequency in prograde circular orbits around a neutron star may be even larger than the orbital frequency ($\omega_z - \Omega_{\rm K} > 0$). As this occurs for astrophysically interesting masses, the effect could have important consequences for models of QPOs (see Section 3), some of which have already been studied for quark stars (Gondek-Rosińska et al., 2014).

Figure 1 shows the scaled orbital and epicyclic frequencies versus gravitational mass (in units of mass of the Sun) at $r = 1.3\,a$ for uniformly rotating neutron stars (left panel) and strange quark stars (right panel) rotating at a fixed frequency $f_{\rm rot} = 600\,$Hz. While at the astrophysically expected masses of $M > M_\odot$ the vertical epicyclic frequency is less than the orbital one ($\omega_z < \Omega_{\rm K}$) for the neutron star modelled with the FPS equation of state (as expected for a metric close to the Kerr one), for lower masses the opposite relation holds, $\omega_z > \Omega_{\rm K}$, as a result of stellar oblateness. However, for higher masses effects of strong gravity, such as frame dragging, dominate the qualitative behaviour of the frequency curves. For the quark star model, the results are qualitatively similar, except that the change in sign of $\omega_z - \Omega_{\rm K}$ takes place at a higher mass value ($M \approx 1.1\,M_\odot$ for the quark star versus $M \approx 0.9\,M_\odot$ for the FPS neutron star).
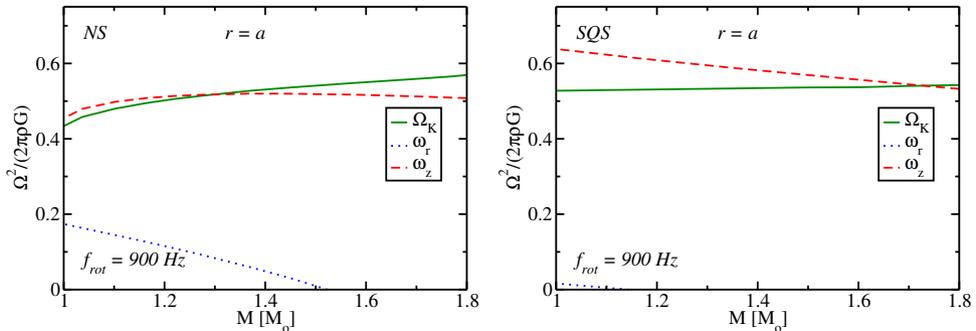
Figure 2. Squares of the orbital and epicyclic frequencies (scaled with $2\pi G\rho$) at the stellar equator ($r = a$) versus gravitational mass for uniformly rotating neutron stars (*left panel*), and strange quark stars (*right panel*), rotating at a fixed frequency $f_{\text{rot}} = 900$ Hz. The solid (*green*) line corresponds to orbital frequency, the dashed (*red*) line to vertical epicyclic frequency and the dotted (*blue*) line to radial epicyclic frequency.

Figure 2 shows the same effects for more rapidly rotating stars ($f_{\text{rot}} = 900$ Hz). The frequencies are presented for $r = a$, i.e. at the stellar equator. Interestingly, the effects of oblateness on the epicyclic frequencies are now seen to qualitatively affect the ordering of the frequencies at typical pulsar masses ($\omega_z > \Omega_{\text{K}}$ for the FPS neutron stars with values of M up to the canonical mass of 1.4 $M_\odot$, while for quark stars this is true up to 1.7 $M_\odot$). In part this is because of the higher rotation rate for these models, i.e. their larger oblateness, and in part because the orbits are closer to the star in Fig. 2 than in Fig. 1 – the higher multipoles decay rapidly with the radial distance, so their effect is more pronounced near the star.

3   DISCUSSION AND CONCLUSIONS

The purpose of this work is to study in GR the influence of oblateness on the orbital and epicyclic frequencies for rapidly rotating neutron stars and to compare the results to analogous results for strange quark stars (Gondek-Rosińska et al., 2014). Surprisingly, we have found that effects of oblateness familiar from Newtonian studies (Kluźniak and Rosińska, 2013), such as decreasing of the radial epicyclic frequency with the stellar rotation rate and the vertical epicyclic frequency exceeding the orbital one ($\omega_z > \Omega_{\text{K}}$), are present for realistic models of rotating neutron stars in general relativity.

Epicyclic frequencies determine the properties of oscillation modes of thin accretion disks (Kato and Fukue, 1980; Wagoner, 1999). One of the most promising modes that may correspond to the observed QPOs, the c-mode, is described by a corrugation of the disk precessing at a frequency close to $\Omega_{\text{K}} - \omega_z$, and it may be present only if $\omega_z < \Omega_{\text{K}}$ (Silbergleit et al., 2001). Our results indicate that for some neutron stars (at least for the FPS equation of state) the latter condition may not hold throughout the inner accretion disk. This could indicate the necessity of revisiting the QPO models. In another class of models (e.g. Kluźniak, 2008), one of the kHz QPOs could correspond directly to motion with the frequency ω_z. One possibility that could now be taken into account is that the higher of the twin kHz QPO frequencies may have a value larger then the orbital frequency ($\omega_z > \Omega_{\text{K}}$).

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