The motion of the test particle near the black hole embedded into dust: The flat space case

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ABSTRACT
In this work we investigate the motion of free particle in the field of strongly gravitating object which is embedded into dust cosmological background. We use newly obtained exact solution of Einstein equations in comoving coordinates for the system under consideration in case of zero spatial curvature. Observable velocity of the particle moving relatively to the observer comoving with cosmological expansion is found from geodesic equations.

Keywords: black hole – cosmological background – LTB solution – test particle – exact solutions of Einstein equations

1 INTRODUCTION

The problem of building a model of the black hole that embedded into space which is not empty but filled with some matter is of great interest in wide set of research directions, including the thermodynamics of black holes (Giddings, 2012), (Firouzjaee and Mansouri, 2012), (Firouzjaee and Ellis, 2015a), (Firouzjaee and Ellis, 2015b), the black hole horizon dynamics (Firouzjaee and Mansouri, 2010), studying the influence of cosmological expansion on the evolution of local objects (Moradi et al., 2010), (Faraoni and Jacques, 2009) etc.

In this work we focus on investigation of the motion of the test particle near the black hole in a cosmological background. This will help in explaining how the effect of cosmological expansion could be relevant for local dynamics near the astrophysical black holes today. Many papers devoted to this problem were appearing over the years (see (Senovilla et al., 1999), (Krasinski, 1997) for brief reviews) but the problem is steel staying a point for active discussions. New exact solutions can help to better understand this problem.

Recently a new exact solution describing the black hole embedded into dust matter was found by means of the mass function method (Korkina and Kopteva, 2012b), (Korkina and Kopteva, 2012a). Using this solution in particular case of flat space we study the motion of the test particle in resulting space–time solving the geodesic equations.

The paper is organized as follows. In section 2 briefly present the idea of the mass function method and our exact solution. In section 3 we write the geodesic equations and solve them for pure radial and orbital motion. In conclusions section we summarize the results.

2 THE SOLUTION

The mass function method is the method for solving the Einstein equations by means of introducing the mass function (Misner and Sharp, 1964) – (Zannias, 1990)

$$m(R,t) = r(R,t)(1 + e^{-\nu(R,t)}\dot{r}^2 - e^{-\lambda(R,t)}r'^2) \tag{1}$$

which is one of four algebraic invariants for the spherically symmetric metric

$$ds^2 = e^{\nu(R,t)}dt^2 - e^{\lambda(R,t)}dR^2 - r^2(R,t)d\sigma^2. \tag{2}$$

Using (1) it is possible to rewrite the Einstein equations in much more simple way

$$m' = \varepsilon r^2 r'; \tag{3}$$

$$\dot{m} = -p_{\|} r^2 \dot{r}; \tag{4}$$

$$2\dot{r}' = \nu' \dot{r} + \dot{\lambda} r'; \tag{5}$$

$$2\dot{m}' = m'\frac{\dot{r}}{r'}\nu' + \dot{m}\frac{r'}{\dot{r}}\dot{\lambda} - 4r\dot{r}r' p_{\perp}; \tag{6}$$

Here and further we use the units were $c = 1$ and $8\pi G = 1$; dot means derivative with respect to t and prime means derivative with respect to R; ε is energy density, $p_{\perp}$ is tangential pressure, and $p_{\|}$ is radial pressure.

In our consideration we use the comoving coordinates, which are known to become synchronous for the dust.

Let us take the metric describing dust distribution in Tolman–Bondi form (Tolman, 1969)

$$ds^2 = dt^2 - \frac{r'^2(R,t)}{f^2(R)}dR^2 - r^2(R,t)d\sigma^2, \tag{7}$$

were $f(R)$ is arbitrary function having sense of total energy in mc^2 units in the shell R.

The Schwarzschild solution as well as Friedman solution are the particular cases of the Tolman–Bondi solution under certain choice of functions $m(R)$, $f(R)$ and $t_0(R)$. Namely $m(R) = r_g$ for the Schwarzschild solution and $m(R) = a_0 R^3$, $f(R) = 1$, $t_0(R) = 0$ for the flat Friedman one.

The mass function has a meaning of total mass in the shell R, so it is additive function. And hence the solution for the Schwarzschild–like black hole in the Tolman–Bondi space–time will be the Tolman–Bondi solution with the mass function

$$m(R) \to r_g + m(R). \tag{8}$$

Thus the solution for the flat Friedman world together with the Schwarzschild–like black hole for the case of expansion takes the form

$$r(R,t) = \left[\pm\frac{3}{2}\sqrt{r_g + a_0 R^3}(t - t_0(R)) \right]^{\frac{2}{3}}. \tag{9}$$

One should notice that this solution does not describe the pure Friedman world but describes the world of Tolman–Bondi with mass function chosen the same as Friedman one.

The metric (7) has two true singularities under $r(R,t) = 0$ and $r'(R,t) = 0$. The energy density in the resulting space–time can be found from the equation (3) regarding the mass function

$$m(R) = (r_g + a_0 R^3). \tag{10}$$

It reads

$$\varepsilon(R,t) = \frac{4a_0 R^2}{(a_0 R^2(3t - 5R) - 2r_g)(t - R)} \tag{11}$$

The expansion starts for each shell R at the moment $t = R$ with infinite energy density and then for each shell the energy density tends to zero with time. The black hole horizon is absent in comoving coordinates, but there is one more singularity which gives divergence of the energy density along $r'(R,t) = 0$. Thus the observer with constant R and φ will always see the overdense region near small R what might be treated as the manifestation of the black hole.

3 THE EQUATION OF MOTION

In this section we consider the motion of the test particle in obtained model with respect to observer comoving with the cosmological expansion. We fix θ coordinate $\theta = \pi/2$, and chose the arbitrary function $t_0(R)$ to be just R to get analogy with Lemaitre solution.

For the solution (9) with mass function (10) and metric

$$ds^2 = dt^2 - r'^2(R,t)dR^2 - r^2(R,t)d\varphi^2 \tag{12}$$

the geodesic equations gives

$$\frac{d^2t}{ds^2} + \dot{r}'r'\Big(\frac{dR}{ds}\Big)^2 + \dot{r}r\Big(\frac{d\varphi}{ds}\Big)^2 = 0 \tag{13}$$

$$\frac{d^2R}{ds^2} + \frac{r''}{r'}\Big(\frac{dR}{ds}\Big)^2 - \frac{r}{r'}\Big(\frac{d\varphi}{ds}\Big)^2 + 2\frac{\dot{r}'}{r'}\frac{dt}{ds}\frac{dR}{ds} = 0 \tag{14}$$

$$\frac{d^2\varphi}{ds^2} + 2\frac{r'}{r}\frac{dR}{ds}\frac{d\varphi}{ds} + 2\frac{\dot{r}}{r}\frac{d\varphi}{ds}\frac{dt}{ds} = 0 \tag{15}$$

For the case when the particle starts from rest with respect to comoving coordinates R, φ one has

$$\frac{dR}{ds} = 0, \quad \frac{d\varphi}{ds} = 0, \quad \frac{dt}{ds} = 1, \tag{16}$$

and hence from the system (13)-(15) it follows that

$$\frac{d^2t}{ds^2} = 0, \quad \frac{d^2R}{ds^2} = 0, \quad \frac{d^2\varphi}{ds^2} = 0. \tag{17}$$

This means that starting from rest the particle is staying in rest respectively to comoving observer and follows the cosmological expansion as all matter averagely do.

If the particle has arbitrary initial velocity in $\theta = \pi/2$ plane then integrating (14) and (15) and using the interval (12) one can obtain the following expressions

$$\frac{ds}{dt} = Bu_1 r'^2 e^{-\frac{u_3}{u_1}\varphi} \tag{18}$$

$$\frac{ds}{dt} = Aru_3 \tag{19}$$

where A and B are arbitrary constants of integration.

$$\left(\frac{ds}{dt}\right)^2 = 1 - (u_1^2 + u_3^2) = 1 - u^2 \tag{20}$$

where $u_1 = r'dR/dt$ is observable velocity of the particle in radial direction, and $u_3 = rd\varphi/dt$ is observable orbital velocity.

Let us consider first the pure radial motion. Then putting $u_3 = 0$ in (18) and (20) we obtain

$$u_1 = \frac{1}{\sqrt{r'^4 + 1}}. \tag{21}$$

For the pure orbital motion one has $u_1 = 0$ and hence

$$u_3 = \frac{1}{\sqrt{r^2 + 1}}. \tag{22}$$

And finally the total velocity reads

$$u^2 = u_1^2 + u_3^2 = \frac{1}{r'^4 + 1} + \frac{1}{r^2 + 1} \tag{23}$$

Thus we have obtained the observable velocity of the test particle which would be measured by the observer being in rest in comoving coordinate frame with his usual instruments. From the expressions for the velocity it follows that even if the test particle has a nonzero initial orbital velocity it will lose its angular momentum with time and moreover will lose its total velocity and will be involved to the cosmological expansion. Yet another situation is possible when the particle falls into the black hole with total velocity tending to the speed of light. The profile of the total velocity in dependence on R and t is represented at fig.1.

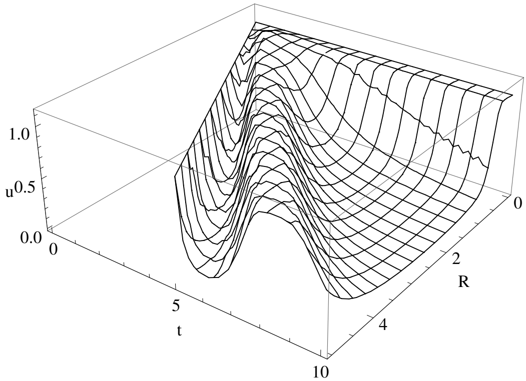

Figure 1. Total velocity profile of the test particle

4 CONCLUSIONS

To conclude one should say that the model under consideration answers the part of questions concerning the behaviour of the test particle moving in the space–time generated by the Schwarzschild–like black hole embedded into the dust matter cosmological background. On the basis of exact solution of the Einstein equations we have obtained the exact analytical expressions for the velocity of the particle, and it turned out that the total velocity of the particle tends to zero with time, that means that the particle will be involved to the cosmological expansion in case it was not traveling towards the center $R = 0$, in this case it would fall into the singularity. The specific character of the coordinate frame makes it impossible to analyze the questions concerning the black hole horizon. And some other interesting problems in this issue were also left for our future consideration.

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