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Comparison of charged particle dynamics around compact object immersed into uniform or dipole magnetic field

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ABSTRACT
We compare charge particle dynamics around magnetized compact object for two different configurations of magnetic field, uniform magnetic field and dipole magnetic field. Comparing charged particle trajectories, position of circular orbits and frequencies of small harmonic oscillations in both magnetic fields, we will try to answer the question if the dipole magnetic field can be approximated by uniform magnetic field at last at small scales.

Keywords: charged particle – black holes – uniform magnetic field – dipole magnetic field

1 INTRODUCTION

Magnetic fields plays very important role in astrophysics as they have been detected and measured in nearly all celestial objects. We consider weak magnetic fields that will satisfy test field approximation - have negligible effect on background spacetime or on the motion of neutral particles. However, for the motion of charged test particles the influence of the magnetic field can be really large. For a charged test particle with charge q and mass m moving in vicinity of a black hole with mass M surrounded by an external asymptotically uniform magnetic field of the strength B, one can introduce a dimensionless quantity b that can be identified as relative Lorenz force (Frolov and Shoom, 2010)

$$b = \frac{|q|BGM}{mc^4}. \tag{1}$$

This quantity can be quite large even for weak magnetic fields due to the large value of the specific charge q/m, and the influence of the magnetic field on the motion of charged particles cannot be neglected even for weak magnetic fields. In our approach the ”charged particle” can represent matter ranging from electron to some charged inhomogeneity orbiting in the innermost region of the accretion disk. The charged particle specific charges q/m for any such structure will then range from the electron maximum to zero.

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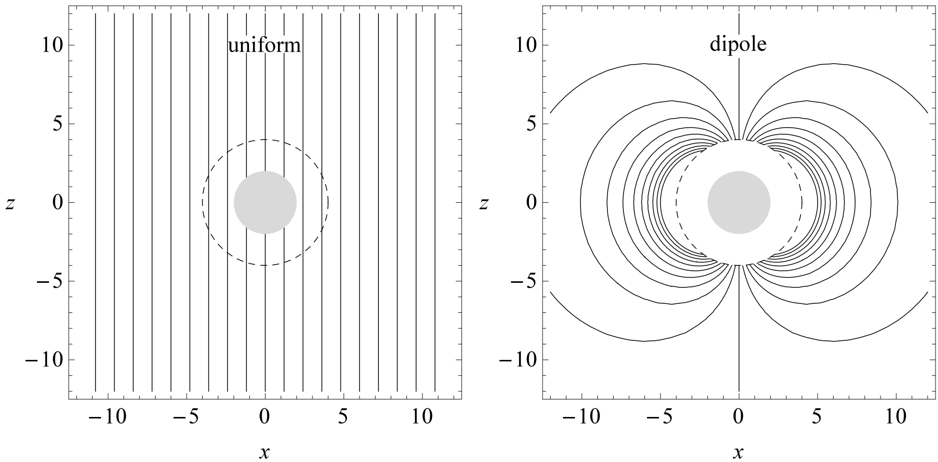

Figure 1. Magnetic field lines for magnetic uniform and dipole fields around Schwarzschild black hole. Uniform magnetic field structure is quite simple - the magnetic field lines are parallel and equally spaced. Dipole magnetic field structure is more complicated and the magnetic field intensity is increasing close to the compact object. Dipole magnetic field is generated by circular current loop with radius $a = 4$, hence magnetic field lines for $2 < r < 4$ are not plotted. The gray disc with radius $r = 2$ represent Schwarzschild black hole horizon, we also plotted dashed circle with radius $r = 4$ representing neutron star surface.

In this paper we will concentrate our attention on two particular cases of magnetized compact object: asymptotically uniform external magnetic field known as Wald solution (Wald, 1974) and relativistic version of dipole magnetic field generated by current loop (Petterson, 1974). Charged particle dynamics in both magnetic filed scenarios has been already widely studied in literature; for example for the uniform magnetic field in (Kopáček et al., 2010; Kološ et al., 2015; Tursunov et al., 2016), while for the dipole magnetic field in (Kovář et al., 2008; Bakala et al., 2010, 2012). The dipole magnetic field configuration is assumed to be much more relevant for neutron stars, while for black holes we can use the uniform field configuration.

Throughout the present paper we use the spacelike signature $(-, +, +, +)$, and the system of geometric units in which $G = 1 = c$.

2 CHARGED PARTICLE DYNAMICS

We describe dynamics of a charged particle with charge $q \neq 0$ in the vicinity of the Schwarzschild black hole embedded in magnetic field, using Lorentz equation and Hamiltonian formalism, and we compare both approachers.

The gravity will enter to the equations of motion through Schwarzschild black hole (with mass M) spacetime line element

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2). \tag{2}$$

We consider first case of magnetic field which is uniform at the spatial infinity, having strength B there. The field is oriented perpendicularly to the equatorial plane of the black hole spacetime. The only nonzero covariant component of the electromagnetic four-vector potential A^μ takes the form (Wald, 1974)

$$A^{\mathrm{U}}_{\phi} = \frac{B}{2}\, g_{\phi\phi}. \tag{3}$$

Dipole magnetic field will be generated by circular current loop with radius $a \geq 2$, located on the surface of compact object in equatorial plane. Outer $r > a$ solution for four-vector potential A^μ in Schwarzschild metric is given by only one nonzero covariant component of the electromagnetic four-vector potential (Petterson, 1974)

$$A^{\mathrm{D}}_{\phi} = -k\left[\ln\left(1 - \frac{2M}{r}\right) + \frac{2M}{r}\left(1 + \frac{M}{r}\right)\right] g_{\phi\phi}, \tag{4}$$

where the term in square brackets is negative for $r > 2$.

Both uniform and dipole magnetic fields are static, have axial symmetry, and the only nonzero covariant component of four-vector potential A^μ can be written as

$$A^{(U,D)}_{\phi} = \text{const.}\, f(r)\, g_{\phi\phi}. \tag{5}$$

For uniform magnetic field the $f(r)$ function is quite simple $f^{\mathrm{U}}(r) = 1$, while for dipole magnetic fields the $f(r)$ function is little bit complicated

$$f^{\mathrm{D}}(r) = -\ln\left(1 - \frac{2M}{r}\right) - \frac{2M}{r}\left(1 + \frac{M}{r}\right). \tag{6}$$

The function $f(r)$ is positive for any $r > 2$.

Hereafter, we put $M = 1$, i.e., we use dimensionless radial coordinate r (and time coordinate t). Cartesian coordinates can be found by the coordinate transformations

$$x = r\cos(\phi)\sin(\theta), \quad y = r\sin(\phi)\sin(\theta), \quad z = r\cos(\theta). \tag{7}$$

The equations of motion for charged particle with charge q and mass m in magnetized Schwarzschild black hole spacetime are given by the Lorentz equation and velocity norming condition

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^{\mu}_{\alpha\beta}u^\alpha u^\beta = \frac{q}{m} g^{\mu\rho} F_{\rho\sigma} u^\sigma, \qquad g_{\mu\nu}u^\mu u^\nu = -1, \tag{8}$$

where $u^\mu = dx^\mu/d\tau$ is the four-velocity of the particle, $\Gamma^{\mu}_{\alpha\beta}$ are Christoffel symbols for Schwarzschild metric 2 and $F_{\mu\nu}$ is tensor of electromagnetic field, given by

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\gamma}\left(g_{\gamma\alpha,\beta} + g_{\gamma\beta,\alpha} - g_{\alpha\beta,\gamma}\right), \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{9}$$

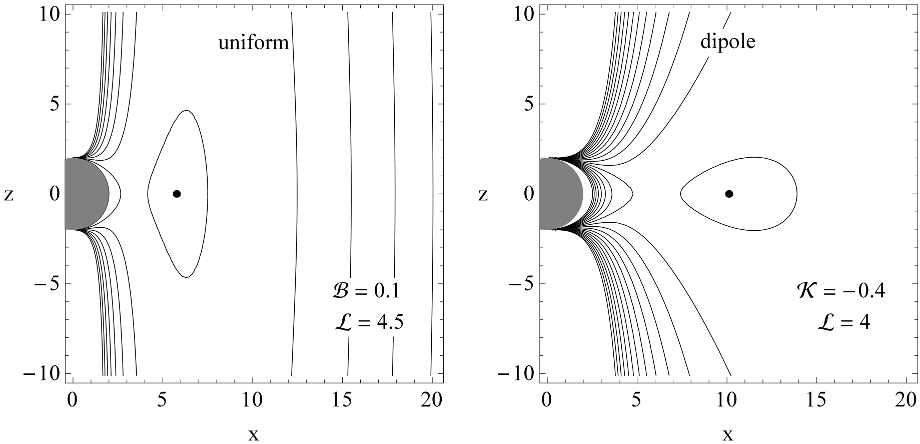

Figure 2. Effective potential for charged particle motion around black hole immersed into uniform (left) or dipole (right) magnetic fields.

The Lorentz equation (8) is set of four second order ordinary differential equations. The main advantage of Lorentz equation is that we can clearly see the Lorentz force $f^{\mu}_{L} = (q/m)F^{\mu\nu}u_{\nu}$ acting on the charged particle.

The equations of motion for charged particle can be also obtained using Hamiltonian formalism

$$\frac{\mathrm{d}x^{\mu}}{\mathrm{d}\zeta} = \frac{\partial H}{\partial \pi_{\mu}}, \quad \frac{\mathrm{d}\pi_{\mu}}{\mathrm{d}\zeta} = -\frac{\partial H}{\partial x^{\mu}}, \qquad H = \frac{1}{2}g^{\alpha\beta}(\pi_{\alpha} - qA_{\alpha})(\pi_{\beta} - qA_{\beta}) + \frac{m^2}{2} = 0, \tag{10}$$

where the kinematical four-momentum $p^{\mu} = mu^{\mu} = \mathrm{d}x^{\mu}/\mathrm{d}\zeta$ is related to the generalized (canonical) four-momentum π^{μ} by the relation $\pi^{\mu} = p^{\mu} + qA^{\mu}$. The affine parameter ζ of the particle is related to its proper time τ by the relation $\zeta = \tau/m$. The Hamiltonian equations (10) is set of eight first order ordinary differential equations. The main advantage of Hamiltonian formalism is the possibility to use very precise numerical integration scheme (symplectic integrator) (Kopáček et al., 2014).

Due to the symmetries of the Schwarzschild spacetime (2) and both uniform and dipole magnetic field (3-4), one can easily find the conserved quantities that are particle energy and axial angular momentum

$$\mathcal{E} = \frac{E}{m} = -\frac{\pi_t}{m} = -g_{tt}u^{t}, \quad \mathcal{L} = \frac{L}{m} = \frac{\pi_{\phi}}{m} = g_{\phi\phi}u^{\phi} + \frac{q}{m}A_{\phi}. \tag{11}$$

Using such symmetries one can rewrite the Hamiltonian (10) in the form

$$H = \frac{1}{2}g^{rr}p_r^2 + \frac{1}{2}g^{\theta\theta}p_{\theta}^2 + \frac{1}{2}g^{tt}E^2 + \frac{1}{2}g^{\phi\phi}(L - qA_{\phi})^2 + \frac{1}{2}m^2 = H_{\mathrm{D}} + H_{\mathrm{P}}, \tag{12}$$

where we separated total Hamiltonian H into dynamical H_{D} (first two terms) and potential H_{P} (last two terms) parts.

We will define new particle and magnetic field parameters: particle specific charge $\tilde{q}$, and magnetic parameters for uniform $\mathcal{B}$ and dipole $\mathcal{K}$ magnetic fileds

$$\tilde{q} = \frac{q}{m} \qquad \mathcal{B} = \frac{qB}{2m}, \qquad \mathcal{K} = -\frac{qk}{m}. \tag{13}$$

Energetic boundary for particle motion can be expressed from the equation (12)

$$\mathcal{E}^2 = V_{\text{eff}}(r,\theta) \quad (\text{for } p_r = p_\theta = 0), \tag{14}$$

We introduced effective potential for charged particle $V_{\text{eff}}(r,\theta)$ by the relation

$$\begin{aligned} V_{\text{eff}}(r,\theta) &\equiv -\, g_{tt}\left[g^{\phi\phi}\left(\mathcal{L} - \tilde{q}A_\phi\right)^2 + 1\right] \\ &= \left(1 - \frac{2}{r}\right)\left[\left(\frac{\mathcal{L}}{r\sin(\theta)} - K\, f(r)\, r\sin(\theta)\right)^2 + 1\right], \end{aligned} \tag{15}$$

where magnetic field constants $K \in \{\mathcal{B},\mathcal{K}\}$ stands $\mathcal{B}$ for uniform and $\mathcal{K}$ for dipole magnetic field, the function $f(r)$ specify the exact magnetic field radial behaviour (5). The effective potential $V_{\text{eff}}(r,\theta)$ combine the influence of gravity potential (first term) with the influence of central force potential given by the specific angular momentum $\mathcal{L}$ and electromagnetic potential energy (terms in square brackets). The effective potential (15) shows clear symmetry $(\mathcal{L}, K) \leftrightarrow (-\mathcal{L}, -K)$, hence from now on we will focus on $\mathcal{L} > 0$ case only. The positive angular momentum of a particle $\mathcal{L} > 0$ means that the particle is revolved in the counter-clockwise motion around the black hole in x-y plane. Example of effective potential $V_{\text{eff}}(r,\theta)$ behaviour can be found in Fig. 2.

For charged particle we distinguish two following situations

\- *Lorentz attractive* (minus configuration), here $K < 0$ - magnetic field and angular momentum parameters have opposite signs and the Lorentz force is attracting the charged particle to the z-axis, towards the black hole.
\+ *Lorentz repulsive* (plus configuration), here $K > 0$ - magnetic field and angular momentum parameters have the same signs and the Lorentz force is repulsive, acting outward the black hole.

For uniform magnetic field, if charge of the particle is taken to be positive $q > 0$, the minus configuration $K < 0$ corresponds to the vector of the magnetic field $\boldsymbol{B}$ pointing downwards, while plus configuration $K > 0$ corresponds to the vector of the magnetic field $\boldsymbol{B}$ pointing upwards the z-axis.

3 CIRCULAR ORBITS AND ISCO

Particles on circular orbits around compact object can form Keplerian accretion disk, with its inner edge given by innermost circular orbit (ISCO). The circular orbit parameters and ISCO position can be determined by examination of effective potential $V_{\text{eff}}(r,\theta)$ function.

The stationary points of the effective potential $V_{\text{eff}}(r,\theta)$ function are given by

$$\partial_r V_{\text{eff}}(r,\theta) = 0, \quad \partial_\theta V_{\text{eff}}(r,\theta) = 0. \tag{16}$$

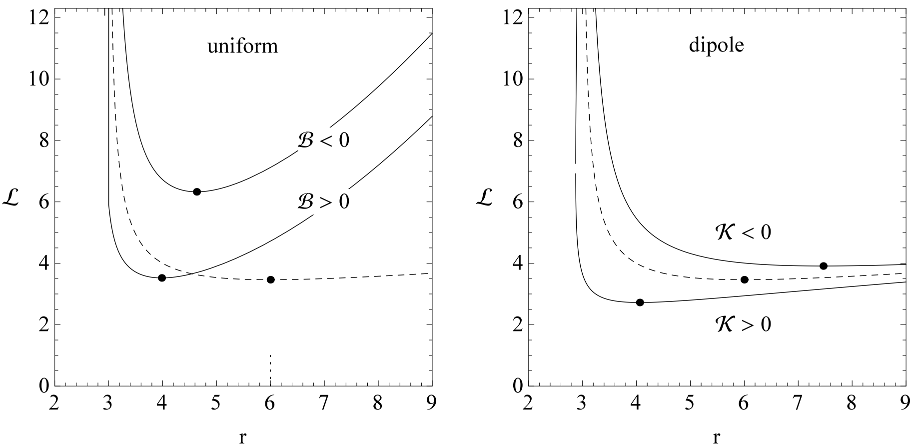

Figure 3. Specific axial angular momentum $\mathcal{L}_{\mathrm{E}}(r)$ for charged particle on circular orbit in magnetic field, as function of radial coordinate r. The non-magnetic case is given as dashed curve, the black point are located at minima of $\mathcal{L}_{\mathrm{E}}(r)$ function and represents radial position of ISCO.

The second equation in the extrema condition (16) has one root at $\theta = \pi/2$. In another words, there is extrema of the $V_{\mathrm{eff}}(r, \theta)$ function located in the equatorial plane. The first equation in the extrema condition (16) leads to a quadratic equation with respect to the specific angular momentum $\mathcal{L}$

$$(r-3)\mathcal{L}^2 + X\mathcal{L} - Y = 0, \tag{17}$$

where functions X, Y are given by

$$X = Kr^2\left[(r-2)rf' + 2f\right], \quad Y = K^2r^4 f\left[(r-2)rf' + (r-1)f\right] + r^2. \tag{18}$$

Real roots of radial coordinate $r > 2$ from eq. (17) determine maxima, minima and inflex points of the $V_{\mathrm{eff}}(r, \theta = \pi/2)$ function. Such extrema give stable (minima) and unstable (maxima) equilibrium positions for the circular particle motion, i.e. stable or unstable circular orbits. The inflex points give the marginally stable circular orbits. The solutions of quadratic equation (17) determine the specific angular momentum $\mathcal{L}_{\mathrm{E}\pm}(r)$ for any circular orbit with radial coordinate r

$$\mathcal{L}_{\mathrm{E}\pm}(r) = \frac{-X \pm \sqrt{X^2 + 4Y(r-3)}}{2(r-3)} \tag{19}$$

The angular momentum $\mathcal{L}_{\mathrm{E}\pm}(r)$ function is plotted for uniform and dipole magnetic fields in Fig. 3. One can clearly see the biggest difference between uniform and magnetic field in equatorial plane. The effect of uniform magnetic became more visible with large radii r, because the magnetic field is constant for any r and does not disappear at infinity. The dipole magnetic field gets weaker as one is departing from the generating current loop

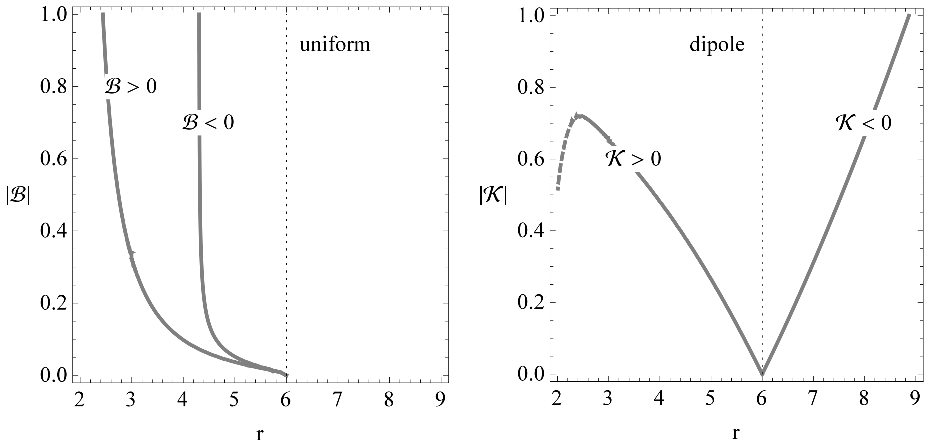

Figure 4. Charged particle ISCO for different values of magnetic field parameters $\mathcal{B}, \mathcal{K}$.

which should be located close to the origin of coordinates but above Schwarzschild horizon, the dipole magnetic field disappear at infinity $r \to \infty$.

The local extrema of the $\mathcal{L}_{E\pm}(r)$ function (19) determine the innermost stable circular orbits (ISCO). The ISCO radial position for charged particle moving around black hole strongly depends on the magnetic field, see Fig. 4. For uniform magnetic field the charged particle ISCO position is decreasing for both attracting ($\mathcal{B} < 0$) or repulsing ($\mathcal{B} > 0$) Lorentz force configurations. Even for relatively small magnetic field parameter $\mathcal{B} = \pm\, 0.05$ the ISCO radii is shifted from from $6M$ to $5M$ in the geometrized units. For dipole magnetic field, the charged particle ISCO position is sifted towards to the black hole for attracting Lorentz force ($\mathcal{K} < 0$), while for repulsive Lorentz force ($\mathcal{K} > 0$) the ISCO position is shifted away from black hole.

4 ANY MAGNETIC FIELD IS UNIFORM AT SMALL SCALES

As numerical simulations shows, realistic magnetic field around compact object with accretion disk will be probably quite complicated. The question arise, if one can substitute this realistic magnetic field by some simple analytic solution, at last for large scales. In this article we will focus on much simpler situation. We will try to substitute dipole magnetic field, as model of complicated magnetic field, with the uniform magnetic field.

Any smooth function can be approximated in neighborhood of some point by linear function using Taylor expansion. We can expand complicated dipole magnetic field into uniform magnetic field at given point, just by assuming to have the the same value of Lorentz force there. We give an example of charged particle moving in dipole and uniform magnetic field where the initial conditions and strength of Lorentz force was set to be the same in both dipole and uniform field cases. In Fig. 5. the particle trajectory will stay close

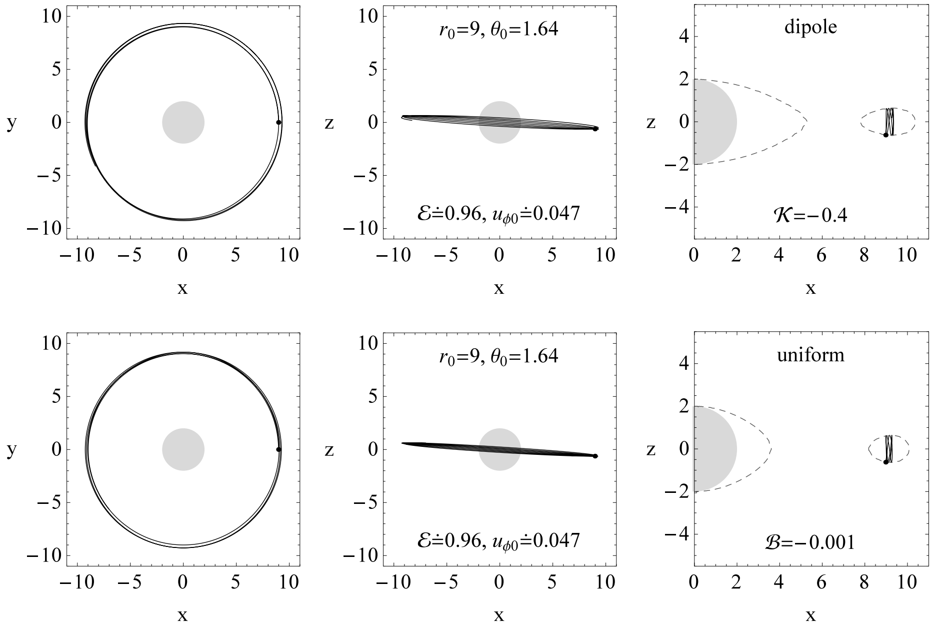

Figure 5. Charged particle trajectory around Schwarzschild black hole with dipole or uniform magnetic filed. Particle initial conditions and strength of Lorentz force are set to be the same in both cases.

to its initial point, because we are close to the effective potential minima. In this case the charged particle trajectory in dipole magnetic field can be well approximated by particle moving in uniform magnetic filed. In Fig. 6. the situation is completely different, since the charged particle in dipole field is exploring large areas below and above equatorial plane, where it will feel the dipole field inhomogeneities. In this situation the approximation by uniform field will valid only for very short time.

Dipole/uniform magnetic field substitution is then possible only for charged particle oscillating around its circular orbit.

5 HARMONIC OSCILLATIONS IN MAGNETIC FIELD

If a charged test particle is slightly displaced from the equilibrium position located in a minimum of the effective potential $V_{\rm eff}(r,\theta)$ at r_0 and $\theta_0 = \pi/2$, corresponding to a stable circular orbit, the particle will start to oscillate around the minimum realizing thus epicyclic motion governed by linear harmonic oscillations. For harmonic oscillations around the minima of the effective potential $V_{\rm eff}$, the evolution of the displacement coordinates $r = r_0 + \delta r, \theta = \theta_0 + \delta\theta$ is governed by the equations

$$\ddot{\delta r} + \omega_{\rm r}^2\, \delta r = 0, \quad \ddot{\delta\theta} + \omega_\theta^2\, \delta\theta = 0, \tag{20}$$

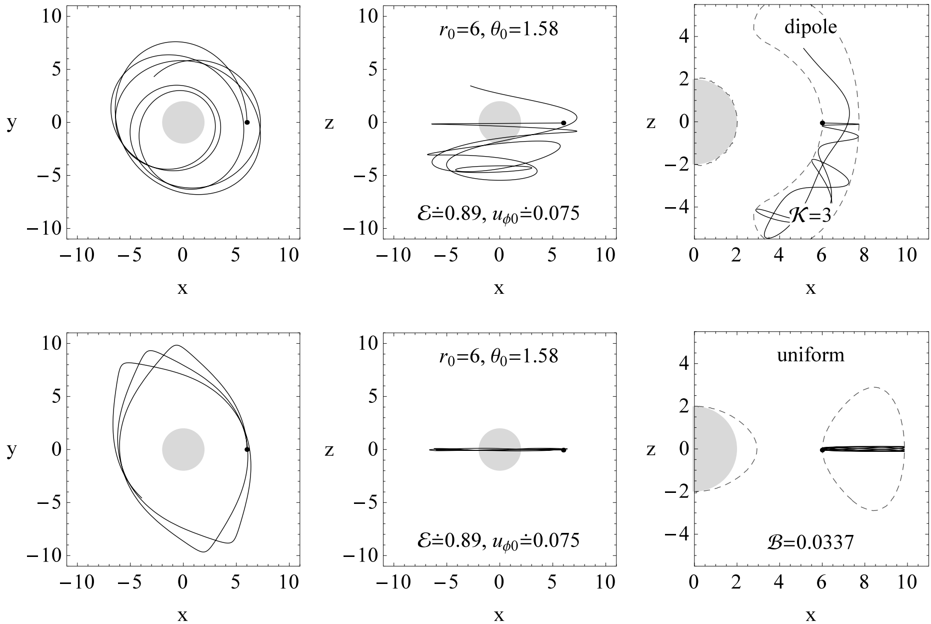

Figure 6. Charged particle trajectory around Schwarzschild black hole with dipole or uniform magnetic filed. Particle initial conditions and strength of Lorentz force are set to be the same in both cases.

where dot denotes derivative with respect to the proper time τ of the particle ($\dot{x} = \mathrm{d}x/\mathrm{d}\tau$), and locally measured angular frequencies of the harmonic oscillatory motion are given by

$$\omega_{\mathrm{r}}^2 = \frac{1}{g_{rr}} \frac{\partial^2 H_{\mathrm{P}}}{\partial r^2}, \quad \omega_{\theta}^2 = \frac{1}{g_{\theta\theta}} \frac{\partial^2 H_{\mathrm{P}}}{\partial \theta^2}, \quad \omega_{\phi} = \frac{\mathrm{d}\phi}{\mathrm{d}\tau} = \mathcal{L} g^{\phi\phi} - K f(r), \tag{21}$$

where we added also the Keplerian (axial) frequency ω_ϕ. We will not put explicit

The locally measured angular frequencies $\omega_{\mathrm{r}}, \omega_\theta$, and ω_ϕ, given by $\omega_\beta = \mathrm{d}\diamond_\beta/\mathrm{d}\tau$ where $\beta \in \{\mathrm{r}, \theta, \phi\}$, are connected to the angular frequencies measured by the static distant observers (in the physical units) by the gravitational redshift transformation

$$\nu_\beta = \frac{1}{2\pi} \frac{c^3}{GM} \frac{\mathrm{d}\diamond_\beta}{\mathrm{d}t} = \frac{1}{2\pi} \frac{c^3}{GM} \frac{\omega_\beta}{-g^{tt}\mathcal{E}(r)}. \tag{22}$$

Behaviour of the frequencies $\nu_{\mathrm{r}}(r), \nu_\theta(r)$ and $\nu_\phi(r)$, as functions of the radial coordinate r, is demonstrated in Fig. 7 for both dipole and uniform magnetic field. For small radii, $r \geq r_{\mathrm{ISCO}}$, we see strong gravitational influence on the angular frequencies in both dipole and uniform cases. For large radii $r \gg r_{\mathrm{ISCO}}$ the influence of the uniform magnetic field is prevailing, while the influence of the dipole magnetic field is fading away and the frequencies are coinciding with non-magnetic case.

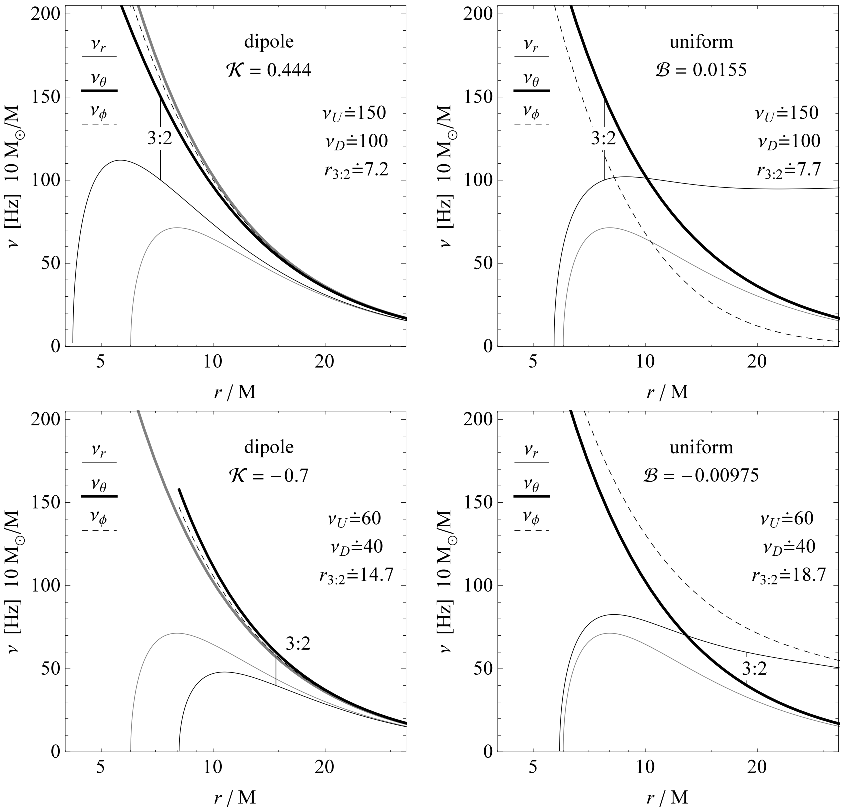

Figure 7. Frequencies $\nu_\theta, \nu_r, \nu_\phi$ of small harmonic charge particle oscillations around magnetized Schwarzschild black hole as measured by static distant observers. One can easily substitute the effect of dipole magnetic field (left figures) with uniform magnetic field (right figures). The black curves are for magnetic case, while the non-magnetic (neutral) case is given by grey curves.

The charged particle oscillations with frequencies $\nu_r(r), \nu_\theta(r)$ and $\nu_\phi(r)$, suggest interesting astrophysical application, related to quasi-periodic oscillations (QPOs) observed in many Galactic Low Mass X-Ray Binaries (LMXB) containing neutron stars or black holes (Bakala et al., 2010; Kološ et al., 2015). According to the observed frequencies of QPOs, which cover the range from few mHz up to 0.5 kHz, different types of QPOs were distinguished. These are the high frequency (HF) and low frequency (LF) QPOs in the timing spectra with frequencies up to 500 Hz and up to 30 Hz, respectively. The HF QPOs are sometimes detected with the twin peaks (upper f_{up} and lower f_{low}) which have frequency ratio close to 3 : 2. The simplest geodesic QPOs model is epicyclic resonance (ER) model (Török et al., 2005), where the two resonant modes are identified to be the radial ν_r and vertical ν_θ epicyclic frequencies

$$f_{up} = \nu_{up} \equiv \nu_\theta, \quad f_{low} = \nu_{low} \equiv \nu_r, \quad \nu_{up} : \nu_{low} = 3 : 2, \tag{23}$$

The frequency commensurability is crucial ingredient of the resonant model, and a particular case of this commensurability occurs for the parametric (internal) resonant phenomena that become strongest in the case of the 3 : 2 frequency ratio

The open question is, if we can once again substitute the influence of complicated magnetic field (dipole), with the simple uniform magnetic field. To fit the observed QPOs frequencies f_{up} and f_{low}, for compact object with mass M, one must change the dipole magnetic field parameter $\mathcal{K}$ in functions $\nu^{\mathrm{D}}_{\mathrm{up}}(r_{\mathrm{D}}, \mathcal{K}, M), \nu^{\mathrm{D}}_{\mathrm{low}}(r_{\mathrm{D}}, \mathcal{K}, M)$ and hence obtain required frequencies for 3:2 resonant radii r_{D}. Frequencies at resonant radii r_{D} for dipole magnetic field with parameter $\mathcal{K}$ can be substituted by frequencies at resonant radii r_{U} for uniform magnetic field $\nu^{\mathrm{U}}_{\mathrm{up}}(r_{\mathrm{U}}, \mathcal{B}, M), \nu^{\mathrm{U}}_{\mathrm{low}}(r_{\mathrm{U}}, \mathcal{B}, M)$ with parameter $\mathcal{B}$ using equations

$$\nu^{\mathrm{D}}_{\mathrm{up}}(r_{\mathrm{D}}, \mathcal{K}, M) = \nu^{\mathrm{U}}_{\mathrm{up}}(r_{\mathrm{U}}, \mathcal{B}, M), \quad \nu^{\mathrm{D}}_{\mathrm{low}}(r_{\mathrm{D}}, \mathcal{K}, M) = \nu^{\mathrm{U}}_{\mathrm{low}}(r_{\mathrm{U}}, \mathcal{B}, M). \tag{24}$$

Examples of frequencies substitution for dipole/uniform magnetic field is show in Fig. 7 for both Lorentz attractive and repulsive cases.

6 CONCLUSIONS

Magnetic field can strongly influence astrophysical processes around compact object. If the specific particle charge q/m is large enough, even weak magnetic field can significantly influence position of Keplerian accretion disc inner edge, the charged particle trajectory and charge particle oscillatory frequencies.

Real magnetic field around compact object will be far away from to be completely uniform, but any magnetic field can be approximated by uniform at last at small scales. Such magnetic field simplification for charged particle motion will work only if the particle trajectory will remain in small region of space. When the particle trajectory will move away from vicinity of its initial position, it will start to feel magnetic field inhomogeneity.

We tested charged particle motion in two magnetic filed configurations: in dipole magnetic field, as representation of complicated filed, and in uniform magnetic field. The charged particle ISCO behave differently for dipole or uniform field: for dipole field the ISCO is increasing or decreasing with the field strength, depending on direction of Lorentz force; for uniform filed is the ISCO always decreasing. Charged particle QPOs frequencies for dipole magnetic field can be substituted by frequencies calculated for uniform magnetic field.

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REFERENCES

Bakala, P., Urbanec, M., Šrámková, E., Stuchlík, Z. and Török, G. (2012), On magnetic-field-induced corrections to the orbital and epicyclic frequencies: paper II. Slowly rotating magnetized neutron stars, *Classical and Quantum Gravity*, **29**(6), 065012.

Bakala, P., Šrámková, E., Stuchlík, Z. and Török, G. (2010), On magnetic-field-induced non-geodesic corrections to relativistic orbital and epicyclic frequencies, *Classical and Quantum Gravity*, **27**(4), 045001.

Frolov, V. P. and Shoom, A. A. (2010), Motion of charged particles near a weakly magnetized Schwarzschild black hole, *Phys. Rev. D* , **82**(8), 084034, arXiv: 1008.2985.

Kološ, M., Stuchlík, Z. and Tursunov, A. (2015), Quasi-harmonic oscillatory motion of charged particles around a Schwarzschild black hole immersed in a uniform magnetic field, *Classical and Quantum Gravity*, **32**(16), 165009, arXiv: 1506.06799.

Kopáček, O., Karas, V., Kovář, J. and Stuchlík, Z. (2010), Transition from Regular to Chaotic Circulation in Magnetized Coronae near Compact Objects, *The Astrophysical Journal* , **722**, pp. 1240–1259, arXiv: 1008.4650.

Kopáček, O., Karas, V., Kovář, J. and Stuchlík, Z. (2014), Application of a symplectic integrator in a non-integrable relativistic system, in Z. Stuchlík, G. Török and T. Pecháček, editors, *RAGtime 10-13: Workshops on black holes and neutron stars*, pp. 123–132.

Kovář, J., Stuchlík, Z. and Karas, V. (2008), Off-equatorial orbits in strong gravitational fields near compact objects, *Classical and Quantum Gravity*, **25**(9), 095011, arXiv: 0803.3155.

Petterson, J. A. (1974), Magnetic field of a current loop around a Schwarzschild black hole, *Phys. Rev. D* , **10**, pp. 3166–3170.

Török, G., Abramowicz, M. A., Kluźniak, W. and Stuchlík, Z. (2005), The orbital resonance model for twin peak kHz quasi periodic oscillations in microquasars, *Astronomy and Astrophysics* , **436**, pp. 1–8.

Tursunov, A., Stuchlík, Z. and Kološ, M. (2016), Circular orbits and related quasi-harmonic oscillatory motion of charged particles around weakly magnetized rotating black holes, *Phys. Rev. D* , **93**(8), 084012, arXiv: 1603.07264.

Wald, R. M. (1974), Black hole in a uniform magnetic field, *Phys. Rev. D* , **10**, pp. 1680–1685.