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Electrostatic effects on the hydrostatic equilibrium of compact stars

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ABSTRACT
We consider electrostatic effect on the fluid distribution of compact star. We modify the energy-momentum tensor including the electric field and current density terms and get a set of hydrostatic equilibrium equations which are an extended version of Tolman-Openheimer-Volkoff (TOV) equations. We expect that solutions of set of hydrostatic equations will lead to a mass-radius relation of the compact star configuration.

Keywords: electrostatic effect – modified energy momentum tensor –charge fluid configuration – charged neutron star

1 INTRODUCTION

Compact stellar objects such as neutron stars are the laboratories for study the physics in extreme conditions, being the crossroad of various disciplines of the contemporary physics. In the recent years, using the multi-wavelength and multi-messenger observations of neutron stars a large amount of data has been collected, stimulating an interest in testing various theories and theoretical models. The direct detection of gravitational waves in the event GW170817, from a binary neutron star merger (Abbott et al., 2017) opened up a new avenue in the investigation of these remarkable objects, on the other hand, restricting the applicability of some of the theories (see, e.g. Radice et al., 2018).

Neutron stars are compact stars with tremendously high densities, in which most of protons and electrons fuse together producing neutrons. However, closer to the surface of the neutron star, where the densities are expected to be less than in the center, some portion of charged particles, like protons and electrons may survive under certain conditions, so the local charge neutrality cannot be imposed (Rotondo et al., 2011). In this contribution we focus on the possibility of the neutron star to have non-negligible electric charge and corresponding electric field, which eventually modifies the conditions for the relativistic hydrostatic equilibrium.

The net charge contribution in case of the neutron stars is often neglected in the literature, justified by lack of astrophysical mechanisms for charging of this object to such values, for which the energy-momentum tensor of electromagnetic field would become comparable with those of the gravitational field of the neutron star. This problem is quite similar to the fact that the Reissner-Nordström spacetime metric for compact objects like black holes is not often used in realistic models. Neglecting the charge also simplifies the equations governing the hydrostatic equilibrium of neutron stars known as the Tolman-Oppenheimer-Volkoff (TOV) equation. However, in addition to the purely conceptual interest in studying charged compact star configurations, one can point out at least two realistic mechanisms of charging of compact stars, which we briefly summarize below.

The first mechanism is based on Arthur Eddington's idea formulated in Eddington (1926). Difference of masses of protons and electrons by a factor of almost 2×10^3 leads to the charge separation in the stellar atmosphere. Therefore, stars should possess a small and positive electric charge to prevent protons and electrons from further separation. Eddington estimated the charge of a star of the order of 100 C per solar mass. Later in 1978, Eddington's idea was generalized by Bally and Harrison (1978), concluding that any macroscopic cosmic body, including galaxies, stars and also the neutron stars bear a positive electric charge of the order of 100 C per solar mass. In this case, the positive charges of cosmic objects are compensated by negatively charged particles, i.e. electrons distributed in the intergalactic and interstellar media. Indeed, for ordinary stars, the density of charge obtained in this mechanism is negligibly small due to large stellar surface. However, due to compactness of neutron stars, having relatively small surface area, the charge density corresponding even to 100 C per solar mass might have some non-negligible impact. Moreover, similar charging mechanism has been recently applied also to black holes (see, e.g. Zajaček et al., 2018; Zajacek and Tursunov, 2019). It has been shown that the charge in case of black hole is not only measurable, but has quite important astrophysical consequences related to the acceleration of cosmic rays (Tursunov et al., 2020a; Tursunov and Dadhich, 2019) and interpretation of observational data of black holes (Tursunov et al., 2020b).

In addition to the above mentioned mechanism, the presence of the charge in neutron stars can be justified by using relativistic approach. Neutron stars are strongly magnetized with the strength of magnetic fields reaching up to 10^{18} G. If the highly magnetized neutron star is rotating (which is often the case, as observed e.g. in pulsars), this causes the induction of non-zero electric charge density, known as the Goldreich-Julian charge density, given by the relation

$$\rho_{\mathrm{GJ}} = \frac{1}{2\pi c}\Omega B, \tag{1}$$

where Ω is the angular velocity of the star (Goldreich and Julian, 1969). The relativistic rotation of a neutron star in the presence of strong and highly ordered magnetic field aligned with the rotation axis induces an electric field as in the case of the classical Faraday's unipolar dynamo, which causes charge separation in the neutron star matter leading to subsequent electric charge density given by Eq. (1).

At a high density of matter of a neutron star, the kinetic energy of electrons may become very high, so that it allows them to escape from the surface of the neutron star. The limit on this charge, thus, should be given by the electro-hydro-static equilibrium equations. Therefore, in both Newtonian and relativistic approaches neutron stars bear non-zero electric charge, motivating us to seek for corresponding modifications of the TOV equations.

The modifications of TOV equations by the presence of an electric charge of the neutron star have been previously studied by several authors. We briefly introduce some of these works. Bekenstein (1971) have shown that the metric corresponding to the spherical distribution of charged perfect fluid matches with the standard exterior Reissner-Nordström spacetime metric. Malheiro et al. (2004); Ray et al. (2006) found that strongly charged neutron star configurations (charge tending to its maximal limit) is possible, although such configuration is likely leads to the collapse of the star and subsequent formation of charged black hole. Bhatia et al. (1969) estimated the electric field on the surface of the star by the value of around $\sim 120\,\mathrm{V/cm}$ by solving the hydrostatic equilibrium equations including electrostatic interaction. Lemos et al. (2015) solved electrically modified TOV equation with an assumption of proportionality of the charge to the energy density distributions. Our approach somewhat follows the work of Lemos et al. (2015) with the difference that we include an additional interaction term in the stress-energy tensor expressed in terms of the four-current density and electromagnetic four-potential. When we neglect the additional term, resulting modified TOV equations match with those obtained by Lemos et al. (2015).

2 RELATIVISTIC BACKGROUND

The metric of the static spherically symmetric star can be written in the following general form

$$\mathrm{d}s^2 = -f(r)c^2\mathrm{d}t^2 + l(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right). \tag{2}$$

Here, $f(r)$ and $l(r)$ are the function of a radial coordinate only, due to the spherical symmetry of the central object. The hydrostatic equation that we derive should be in accordance with the Einstein-Maxwell equations, which read

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \qquad \nabla_\nu F^{\mu\nu} = \frac{4\pi}{c}j^\mu, \tag{3}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Faraday-Maxwell tensor written in the terms of the electromagnetic potential A_μ. Here $j^\mu = \rho_c u^\mu$ is the four-current of an electromagnetic field, where ρ_c is the charge density and u^μ is the four-velocity normalized by the condition $u_\mu u^\mu = -c^2$, which we can write as $u_\mu = (c\sqrt{-g_{tt}}, 0, 0, 0)$ in the static case. Note, that we use physical units throughout the paper.

We assume a static neutron star configuration with non-vanishing net charge density of the star, so that the four-current can be written as

$$j^\mu = \left(c\rho_c\sqrt{-g^{tt}}, 0, 0, 0\right). \tag{4}$$

2.1 Energy-momentum tensor

The total energy-momentum tensor can be represented as the sum of two terms

$$T_{\mu\nu} = T_{\mu\nu}^{M} + T_{\mu\nu}^{EM}, \tag{5}$$

where $T_{\mu\nu}^{M}$ corresponds to the energy-momentum tensor of a matter and $T_{\mu\nu}^{EM}$ is an electromagnetic energy-momentum tensor. In the present paper we consider a matter to be a perfect fluid with the total mass density ρ, pressure P, and four-velocity u_μ. Then the matter energy-momentum tensor $T_{\mu\nu}^{M}$ takes the form

$$T_{\mu\nu}^{M} = \left(\rho + P/c^2\right) u_\mu u_\nu + P g_{\mu\nu}. \tag{6}$$

The energy-momentum tensor responsible for the electromagnetic part can be found by variation of the action

$$S = \frac{1}{c} \int \mathcal{L}_{EM} \sqrt{-g} \mathrm{d}^4 x, \tag{7}$$

with respect to the metric, where the Lagrangian is given by

$$\mathcal{L}_{EM} = \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + A_\mu j^\mu. \tag{8}$$

Here the first term of Eq. (8) is the pure contribution due to the electromagnetic field and the second term is responsible for the interaction of charged particles with an electromagnetic field. For the interaction term of the Lagrangian, one can find the corresponding energy-momentum tensor by using the Noether's theorem (Noether, 1971). It basically states that for each differentiable symmetry of the action of a physical system associates a conservation law. Generalization of the Noether's theorem to non-local field theories was studied in Krivoruchenko and Tursunov (2019). For the first term, describing the field, the energy-momentum tensor is calculated in a standard manner by variation of the action Eq. (7) with respect to $g_{\mu\nu}$ and putting on the boundaries $\delta g^{\mu\nu} = 0$, which gives

$$\frac{1}{2} \sqrt{-g} T_{\mu\nu} = \frac{\partial \sqrt{-g} \mathcal{L}}{\partial g^{\mu\nu}} - \frac{\partial}{\partial x^\lambda} \frac{\partial \sqrt{-g} \mathcal{L}}{\partial g^{\mu\nu}_{,\lambda}}. \tag{9}$$

Thus, one can write the total energy-momentum tensor for electromagnetic field with the interaction in the following form

$$T_{\mu\nu}^{EM} = \frac{1}{4\pi} \left(F_{\mu}^{\cdot\gamma} F_{\nu\gamma} - \frac{1}{4} g_{\mu\nu} F_{\gamma\beta} F^{\gamma\beta} \right) + A_\mu j_\nu. \tag{10}$$

One should note, that the symmetric property of the energy-momentum tensor requires that in the last term of Eq. (10) both A_μ and j_ν correspond to the same source.

Due to the spherical symmetry of the star, only nonzero component of an electric field should be E_r which implies that the nonzero components of a tensor of electromagnetic field are $F^{tr} = -F^{rt}$. Thus, the non vanishing components of the total energy-momentum tensor Eq. (5) take the form

$$T_{tt} = \rho c^2 f(r) + \frac{1}{8\pi} l^{-1}(r) F_{rt}^2 + A_t(r) j_t(r), \tag{11}$$

$$T_{rr} = P l(r) - \frac{1}{8\pi} f^{-1}(r) F_{rt}^2, \tag{12}$$

$$T_{\theta\theta} = r^2 \left(P + \frac{1}{8\pi} f^{-1}(r) l^{-1}(r) F_{rt}^2 \right), \tag{13}$$

$$T_{\phi\phi} = r^2 \left(P + \frac{1}{8\pi} f^{-1}(r) l^{-1}(r) F_{rt}^2 \right) \sin^2\theta. \tag{14}$$

Now, let us rewrite the Maxwell equation given in Eq. (3) as

$$\partial_\nu \left(\sqrt{-g} F^{\mu\nu} \right) = \frac{4\pi}{c} j^\mu \sqrt{-g} \,. \tag{15}$$

Here g is the determinant of metric tensor $g_{\mu\nu}$. If we solve the Maxwell equation then we get

$$\begin{aligned} \partial_r \left(\sqrt{l(r) f(r)} r^2 F^{tr} \right) &= 4\pi \rho_c r^2 \sqrt{l(r)}, \\ \Rightarrow F^{tr} &= \frac{Q(r)}{r^2 \sqrt{l(r) f(r)}}. \end{aligned} \tag{16}$$

The above equation can be rewritten as,

$$\frac{\mathrm{d}A_t(r)}{\mathrm{d}r} = \frac{Q(r)}{r^2} \sqrt{f(r) l(r)} \,, \tag{17}$$

where

$$\frac{\mathrm{d}Q(r)}{\mathrm{d}r} = 4\pi r^2 \rho_c \sqrt{l(r)} \,. \tag{18}$$

3 HYDROSTATIC EQUATIONS

Let us assume the function $l(r)$ in the spacetime metric satisfies the following relation

$$\frac{1}{l(r)} = 1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}, \tag{19}$$

which coincides with the external Reissner-Nordström metric.

Now we try to find the differential equation concerning mass of the stellar object considering F^{tr} from Eq. (16). Einstein equation for $G_{tt} = (8\pi G/c^4) T_{tt}$ is given by

$$\frac{f(r) \left(r l'(r) + l^2(r) - l(r) \right)}{r^2 l^2(r)} = \frac{8\pi G}{c^4} f(r) \rho(r) c^2 + \frac{G f(r) Q^2(r)}{c^4 r^4} + \frac{8\pi G}{c^4} A_t(r) j_t(r) \,. \tag{20}$$

Using the form of metric as per expression Eq. (19) we get,

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho(r) + \frac{Q(r)}{c^2 r}\frac{\mathrm{d}Q(r)}{\mathrm{d}r} + \frac{4\pi r^2 A_t(r) j_t(r)}{c^2 f(r)} \,. \tag{21}$$

Taking the solution of Maxwell equation, F^{tr} from Eq. (16), we write Einstein equation for $G_{rr} = (8\pi G/c^4)T_{rr}$ as

$$\frac{r f'(r) - f(r) l(r) + f(r)}{r^2 f(r)} = \frac{8\pi G}{c^4} l(r) P(r) - \frac{G l(r) Q^2(r)}{c^4 r^4} \,. \tag{22}$$

By manipulating above equation we get,

$$\frac{\mathrm{d}f(r)}{\mathrm{d}r} = \frac{8\pi G}{c^4} r f(r) l(r) P(r) - \frac{f(r)}{r} + \frac{f(r) l(r)}{r} - \frac{G f(r) l(r) Q^2(r)}{c^4 r^3} \,. \tag{23}$$

Now we are interested to see the radial dependence of pressure inside the star. For this we introduce the energy-momentum conservation equation as

$$\nabla_\mu T^{\mu\nu} = 0 \,. \tag{24}$$

For $\nu = 1$ we get,

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{\left(\rho(r)c^2 + P(r)\right) f'(r)}{2 f(r)} + \frac{Q(r)}{4\pi r^4}\frac{\mathrm{d}Q(r)}{\mathrm{d}r} - \frac{A_t(r) j_t(r) f'(r)}{2 f^2(r)} \,. \tag{25}$$

Eqs. (17), (18), (21), (23), and (25) are the set of five governing equations which we have to solve simultaneously in order to obtain mass-radius relation.

3.1 Set of equations to be solved

Using Eq. (19) we can simplify Eq. (23) as follows

$$\frac{\mathrm{d}f(r)}{\mathrm{d}r} = \frac{f(r)\left(\frac{8\pi G r P(r)}{c^4} + \frac{2Gm(r)}{c^2 r^2} - \frac{2GQ^2(r)}{c^4 r^3}\right)}{\left(1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}\right)}. \tag{26}$$

From Eq. (4) we can write

$$j_t = c\rho_c(r) g_{tt}\sqrt{-g^{tt}} = -c\rho_c(r)\sqrt{f(r)} \,, \tag{27}$$

and substituting Eqs. (18), (26) and (27) into the pressure equation Eq. (25) we get

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{\left(\rho(r)c^2 + P(r)\right)\left(\frac{4\pi G r P(r)}{c^4} + \frac{Gm(r)}{c^2 r^2} - \frac{GQ^2(r)}{c^4 r^3}\right)}{\left(1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}\right)} + \rho_c \frac{Q(r)/r^2}{\sqrt{1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}}} + \frac{c\rho_c A_t(r)\left(\frac{4\pi G r P(r)}{c^4} + \frac{Gm(r)}{c^2 r^2} - \frac{GQ^2(r)}{2c^4 r^3}\right)}{\sqrt{f(r)}}. \tag{28}$$

Below we summarize a set of equations which we have to solve in order to describe the configuration

$$\frac{\mathrm{d}A_t(r)}{\mathrm{d}r} = \frac{Q(r)}{r^2}, \tag{29}$$

$$\frac{\mathrm{d}Q(r)}{\mathrm{d}r} = \frac{4\pi r^2 \rho_c}{\sqrt{1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}}}, \tag{30}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho(r) + \frac{4\pi r \rho_c Q(r)}{c^2 \sqrt{1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}}} - \frac{4\pi r^2 \rho_c A_t(r)}{c\sqrt{f(r)}}, \tag{31}$$

$$\frac{\mathrm{d}f(r)}{\mathrm{d}r} = \frac{f(r)\left(\frac{8\pi GrP(r)}{c^4} + \frac{2Gm(r)}{c^2 r^2} - \frac{2GQ^2(r)}{c^4 r^3}\right)}{\left(1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}\right)}, \tag{32}$$

$$\begin{aligned}\frac{\mathrm{d}P(r)}{\mathrm{d}r} =& -\frac{\left(\rho(r)c^2 + P(r)\right)\left(\frac{4\pi GrP(r)}{c^4} + \frac{Gm(r)}{c^2 r^2} - \frac{GQ^2(r)}{c^4 r^3}\right)}{\left(1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}\right)} \\ &+ \rho_c \frac{Q(r)/r^2}{\sqrt{1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{c^4 r^2}}} \\ &+ \frac{c\rho_c A_t(r)\left(\frac{4\pi GrP(r)}{c^4} + \frac{Gm(r)}{c^2 r^2} - \frac{GQ^2(r)}{2c^4 r^3}\right)}{\sqrt{f(r)}}.\end{aligned} \tag{33}$$

3.2 Density profile of mass and charge

We have five differential equations to be solved to find $A_t(r)$, $Q(r)$, $m(r)$, $f(r)$, $P(r)$, $\rho(r)$ and $\rho_c(r)$. So we have to get rid of two unknowns to close the system. At first we calculate for constant density inside distribution i.e.

$$\rho(r) = \text{constant}\,. \tag{34}$$

We assume

$$\rho_c(r) = \beta\rho(r)\,, \tag{35}$$

where β is a dimensionless parameter describing the charge fraction in the distribution.

As now we know $\rho(r)$ and $\rho_c(r)$ from Eqs. (34) and (35), we finally have five equations for five unknowns. Therefore the system is closed.

3.3 Initial and boundary conditions

At the center of the star we can consider that $m(0) = 0$, $Q(0) = 0$, $A_t(0) = 0$ and $l(0) = 1$. We also consider that $P(0) = P_0$, $\rho(0) = \rho_0$ and $\rho_c(0) = \rho_{c0}$ where P_c is the central pressure, ρ_0 is the central mass density and ρ_{c0} is the central charge density.

We have to consider the pressure of the distribution vanish at the surface i.e. $P(R) = 0$, where R is the radius of the star. Apart from that it must be taken into account that the electric field at infinity is zero, i.e. $r \to \infty$, $A_t \to 0$ and the spacetime is asymptotically flat which means $r \to \infty$, $f \to 1$.

Following these initial and boundary conditions if we solve system of equations numerically then we can find mass-radius relation and maximum mass that can be supported by this configuration. We leave the numerical calculations for further studies.

4 SUMMARY

We revisited the problem of charged neutron star which might be realistic and astrophysically relevant. We have derived modified TOV Eqs. (29) - (33), governing quantities of our interest. These equations together with Eqs. (34) and (35) can be simultaneously solved numerically, which we will complete in the future work, where we also plan to use realistic equations of state.

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