Particle ionization near a weakly charged black hole

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ABSTRACT
In many astrophysical scenarios, the charge of the black hole is often neglected due to unrealistically large values of the charge required for the Reissner-Nordström spacetime metric. However, black holes may possess a small electric charge due to various selective accretion mechanisms. In this paper we investigate the process of ionization of a neutral particle in the vicinity of a weakly charged non-rotating black hole and calculate the energy of an ionized particle in a chosen scenario. High energy obtained by a charged particle after the ionization process can serve as a distinguishing signature of the weakly charged black holes.

Keywords: Black hole – Electric charge – Ionization – Particle acceleration

1 INTRODUCTION

Recently, it has been pointed out that the ionization or decay of neutral particles in the vicinity of a rotating Kerr black hole immersed into an external magnetic field can lead to the acceleration of ionized particles to ultra high energies, with the Lorentz γ-factors of particles exceeding 10^{12} (Tursunov et al., 2020a; Tursunov and Dadhich, 2019; Stuchlík et al., 2020). The formalism of accelerating mechanism is based on the magnetic Penrose process (Wagh et al., 1985; Parthasarathy et al., 1986) in its ultra-efficient regime (Tursunov et al., 2020a), in which the energy of an ionized particle drives away the rotational energy of the black hole through electromagnetic interaction. It was claimed that the driving engine of the process is in the induced electric field generated by the rotation of the black hole in the external magnetic field.

In this contribution we investigate whether the acceleration of ionized particles can be achieved in a more simplified setting, namely, in the vicinity of a non-rotating Schwarzschild black hole with a radial test electric field. By a *test* electric field we denote the field, whose energy-momentum tensor can be neglected in the description of the gravitational field of the black hole. This implies that the electric field influences the dynamics of charged particles only, being negligible for the geodesics of neutral particles.

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Such a simplified setup is motivated by the following reasons. First of all, the *no-hair theorem* states that the spacetime around black holes can be fully described by at most three metric parameters - black hole mass, spin and electric charge. The later is usually neglected in astrophysical scenarios, justified by unrealistically large values of the charge required for its visible effect on the spacetime metric. Indeed, one can compare the gravitational radius of a black hole with the characteristic length of the charge Q_{G} of the Reissner-Nordström black hole, which gives the maximum charge value

$$\sqrt{\frac{Q_{\mathrm{G}}^2 G}{c^4}} = \frac{2GM}{c^2}, \tag{1}$$

$$\Rightarrow Q_{\mathrm{G}} = 2G^{1/2}M \approx 10^{31}\frac{M}{10M_{\odot}}\ \mathrm{Fr}. \tag{2}$$

This value of the charge is unattainable in any known astrophysically relevant scenario. Thus, the Reissner-Nordström spacetime metric is interesting, but astrophysically not viable.

On the other hand, there exist several astrophysical mechanisms based on a selective accretion, in which a black hole can be weakly charged. Since protons are about 1836 times more massive than electrons, the balance between the gravitational and Coulombic forces for the particles close to the surface of the compact object is obtained when the black hole acquires a positive net electric charge of the order of $Q \sim 3 \times 10^{11}$Fr per solar mass (Zajacek and Tursunov, 2019; Bally and Harrison, 1978). Moreover, matter surrounding black hole can be ionized and charged by the irradiating photons taking away some electrons (Weingartner et al., 2006). Perhaps the most famous mechanism of charging of black holes is based on the solution by Wald (1974), in which the charge is induced by the twisting of magnetic field lines due to the frame-dragging effect. As a result, both the black hole and surrounding magnetosphere should acquire an equal and opposite charge of the order of $Q \sim 10^{18}$Fr per solar mass (see, e.g. Tursunov et al., 2020b). In all cases, the charge of the black hole is much weaker than the maximal value (2) by many orders of magnitude (see, e.g. for the Galactic center supermassive black hole in Zajaček et al., 2018), therefore, the gravitational effect of the charge on the spacetime metric can be rightly neglected. One should also note that our consideration of a weakly charged Schwarzschild black hole in the current paper is reasonable and well justified unless the value of the black hole charge is comparable with its maximum limit (2).

Below we will show that even such a weak electric field ($Q \ll Q_{\mathrm{G}}$) cannot be neglected in the description of the motion of charged particles. Moreover, it plays a crucial role in the mechanism of acceleration of ionized particles. Hereafter we use the signature $(-,+,+,+)$, and the system of geometric units, in which $G = 1 = c$, unless the units are given explicitly in physical units.

2 DYNAMICS OF A CHARGED PARTICLE

2.1 Background setup & equations of motion

The Schwarzschild solution of the Einstein's field equations, corresponding to a spherically symmetric spacetime metric reads

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3}$$

where $f(r)$ is a lapse function parametrized by the black hole mass M as follows

$$f(r) = 1 - \frac{2M}{r}. \tag{4}$$

Let us assume the presence of the radial electric field with a corresponding small electric charge Q at the center. Then, the only non-zero covariant component of the electromagnetic potential $A_\mu = (A_t, 0, 0, 0)$ takes the following simple form

$$A_t = -\frac{Q}{r}. \tag{5}$$

The anti-symmetric tensor of the electromagnetic field $F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$ has the only one independent nonzero component

$$F_{tr} = -F_{rt} = -\frac{Q}{r^2}. \tag{6}$$

Let us now consider the motion of a charged particle of mass m and charge q in the combined background of gravitational and electric fields. The motion of a charged particle is governed by the Lorentz equation in curved spacetime

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\alpha\beta}u^\alpha u^\beta - \frac{q}{m}F^\mu{}_\nu u^\nu = 0, \tag{7}$$

where u^μ is the four-velocity of the particle, τ is the proper time of the particle and $\Gamma^\mu_{\alpha\beta}$ – Christoffel symbols.

Due to symmetries of the background Schwarzschild metric, one can introduce two integrals of motion, corresponding to temporal and spatial components of the canonical four-momentum of the charged particle

$$\frac{P_t}{m} = -\mathcal{E} \equiv -\frac{E}{m} = u_t - \frac{qQ}{mr}, \tag{8}$$

$$\frac{P_\phi}{m} = \mathcal{L} \equiv \frac{L}{m} = u_\phi, \tag{9}$$

where $\mathcal{E}$ and $\mathcal{L}$ denote specific energy and specific angular momentum of the charged particle. Since both gravitational and electric fields are spherically symmetric and there is no preferred plane of the motion, one can fix the motion of the charged particle to the equatorial plane ($\theta = \pi/2$), without loss of generality. Thus, three non-vanishing components of

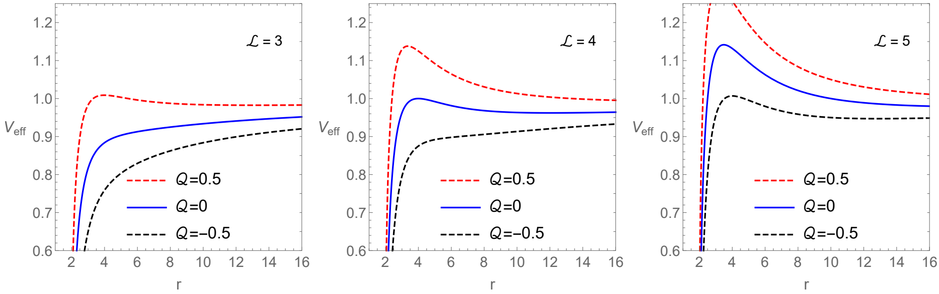

Figure 1. The radial dependence of the effective potential V_{eff} for a charged particle around weakly charged non-rotating black hole in the equatorial plane $\theta = \pi/2$ for different values of the parameters $\mathcal{L}$ and $\mathcal{Q}$.

the equation of motion (7) can be found in the form

$$\frac{\mathrm{d}u^t}{\mathrm{d}\tau} = \frac{u^r \left[\mathcal{Q} r - 2M\left(e r + \mathcal{Q}\right)\right]}{r\left(r-2M\right)^2}, \tag{10}$$

$$\frac{\mathrm{d}u^r}{\mathrm{d}\tau} = \frac{e\mathcal{Q}}{r^2} + \frac{\mathcal{L}^2\left(r-2M\right)}{r^4} - \frac{M\left[e^2 - (u^r)^2\right]}{r\left(r-2M\right)}, \tag{11}$$

$$\frac{\mathrm{d}u^\phi}{\mathrm{d}\tau} = -\frac{2\,\mathcal{L}\,u^r}{r^3}, \tag{12}$$

$$\text{where} \quad e = \mathcal{E} - \frac{qQ}{mr}. \tag{13}$$

Equations (10) - (12) are ordinary differential equations, which can be easily solved numerically.

2.2 Effective potential

Using the normalization condition for a massive particle $u^\mu u_\mu = -1$, one can derive the effective potential for the charged particle moving around a weakly charged Schwarzschild black hole in the form

$$V_{eff}(r) = \frac{\mathcal{Q}}{r} + \sqrt{f(r)\left(1 + \frac{\mathcal{L}^2}{r^2}\right)}, \tag{14}$$

where $\mathcal{Q} = Qq/m$ is a parameter characterizing the electric interaction between the charges of the particle and black hole. Without loss of generality, we set the mass of the black hole to be equal to unity, i.e. $M = 1$.

Since the right hand side of the effective potential (14) is always positive one can distinguish two qualitatively different situations depending on the sign of the parameter $\mathcal{Q}$. When $\mathcal{Q} > 0$, the charges of the particle and black hole have the same sign, so the electric interaction is repulsive. In the opposite case, when $\mathcal{Q} < 0$, the charges of the particle and black hole have different signs, so the electric interaction is attractive. The term $\mathcal{L}^2$ under the root of Eq.(14) means that the clockwise and counter-clockwise directions of the motion are equivalent.

The radial profile of the effective potential is shown in Figure 1. One can see that the effect of the charge parameter Q is similar to those of the angular momentum $\mathcal{L}$, i.e. increasing (or decreasing) both parameters Q and $\mathcal{L}$ one can increase (or decrease) the value of the effective potential. It is interesting to note that taking into account the parameter Q can mimic the effect of angular momentum (compare, e.g. red curve in the middle plot with a very similar blue curve on the right plot of the Figure 1).

The stationary points of the effective potential $V_{eff}(r)$ are given by the equation

$$\partial_r V_{eff}(r) = 0. \tag{15}$$

Note that in the case of a weakly charged Schwarzschild black hole all the local extrema of the effective potential V_{eff} are located in the equatorial plane $\theta = \pi/2$. Eq. (15) leads to a polynomial equation of the fourth order in the radial coordinate

$$r^2(J-1) + \mathcal{L}^2(r-3) = 0, \tag{16}$$

$$\text{where} \quad J = \frac{Q}{r}\sqrt{\frac{(r-2)\left(\mathcal{L}^2 + r^2\right)}{r}}. \tag{17}$$

A solution of the equation (16) has four roots of $\mathcal{L}$ with two of them being independent

$$\mathcal{L}^2_{\pm} = \frac{r}{(r-3)^2}\left(-Q^2 - 3r + \frac{Q^2 r}{2} + r^2 \pm Q\sqrt{Q^2 - 12r + 4r^2}\left(1 - \frac{r}{2}\right)\right), \tag{18}$$

2.3 Angular velocity measured at infinity

Noticing that in the equatorial plane the four velocity takes the form $u^\alpha = u^t(1, v, 0, \Omega)$, where $v = dr/dt$, $\Omega = d\phi/dt$ and using the normalization condition $u^\alpha u_\alpha = -k$, where $k = 1$ for massive particle and $k = 0$ for massless particle, we can obtain the following equation

$$(u^t)^2\left(f^{-1}(r)v^2 - f(r) + \Omega^2 r^2\right) = -k. \tag{19}$$

Simplifying the equation above, we can easily derive an equation for angular velocity measured by a static observer at infinity $\Omega = d\phi/dt$

$$\Omega = \pm\frac{1}{u_t r}\sqrt{(u_t)^2\left(f(r) - f^{-1}(r)v^2\right) - kf^2(r)}. \tag{20}$$

Allowed values of Ω are limited to

$$\Omega_- \leq \Omega \leq \Omega_+, \quad \Omega_\pm = \pm\frac{\sqrt{f(r)}}{r}. \tag{21}$$

corresponding to the photon motion.

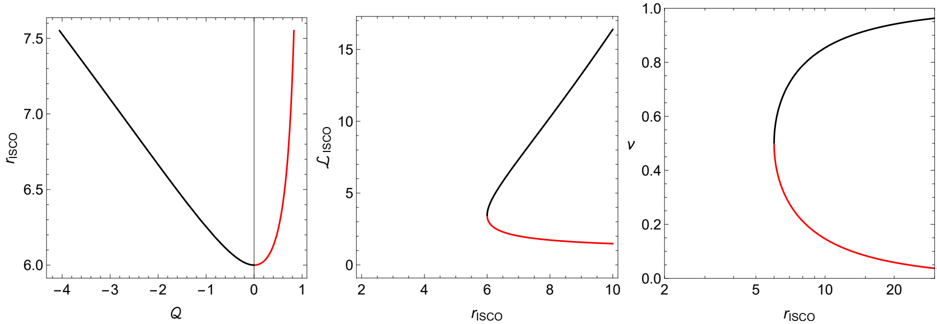

Figure 2. Left: The position of the ISCO of the charged particle in the dependence on the charge parameter Q. Middle: Angular momentum of the charged particle at ISCO against ISCO position. Right: velocity of the charged particle at ISCO. In all plots, the red lines correspond to the positive charge parameter $Q > 0$, while the black curves correspond to the negative charge parameter $Q < 0$.

2.4 Innermost stable circular orbit

An innermost stable circular orbit (ISCO) in the Schwarzschild spacetime is located at $r_{ISCO} = 6M$. In the case when the electric charge is included, it will be shifted outwards from $6M$. The local extremum of the function $\mathcal{L}_{\pm}$ determines the ISCO, namely, its radius, angular momentum and energy. ISCO can also be found from the condition of $\partial_r^2 V_{eff}(r, \mathcal{L}, Q) = 0$, which gives

$$\mathcal{L}^2 r^2 (J(r-2)+2) + r^4 (J(r-2) - r + 3) + \mathcal{L}^4((r-3)r+3) = 0. \tag{22}$$

Solving this equation with respect to r gives us four solutions for the ISCO with only two of them being real and independent. One can also calculate the velocity v of the charged particle at the ISCO, which is given by the formula

$$v = \sqrt{\frac{1}{1 + \frac{r_{isco}^2}{\mathcal{L}_{isco}^2}}}. \tag{23}$$

Dependence of the ISCO position r_{isco} on the charge parameter Q and the change of the values of $\mathcal{L}_{isco}$ and v on the ISCO position are shown in Figure 2. ISCO is increasing for both positive and negative Q. Similar results have been also obtained recently by Hackstein and Hackmann (2020), where the ISCO in a similar setting is properly discussed.

3 THE ENERGY OF THE IONIZED PARTICLE

3.1 Conservation laws

Let us now consider the decay of a particle 1 into two fragments 2 and 3 close to the horizon of a weakly charged Schwarzschild black hole at the equatorial plane. One can write the following conservation laws before and after decay

$$E_1 = E_2 + E_3, \quad L_1 = L_2 + L_3, \quad q_1 = q_2 + q_3, \tag{24}$$

$$m_1 \dot{r}_1 = m_2 \dot{r}_2 + m_3 \dot{r}_3, \quad m_1 \geqslant m_2 + m_3, \tag{25}$$

where dot indicates derivatives with respect to the particle's proper time τ. Using the above conservation laws, one can find the equation

$$m_1 u_1^\phi = m_2 u_2^\phi + m_3 u_3^\phi. \tag{26}$$

Noticing that $u^\phi = \Omega u^t = \Omega e/f(r)$, where $e_i = (E_i + q_i A_t)/m_i$, with $i = 1, 2, 3$ indicating the particle's number, the equation (26) will take the following from

$$\Omega_1 m_1 e_1 = \Omega_2 m_2 e_2 + \Omega_3 m_3 e_3. \tag{27}$$

Solving the above equation with respect to the energy of one of the fragments, e.g. E_3 we find

$$E_3 = \frac{\Omega_1 - \Omega_2}{\Omega_3 - \Omega_2} (E_1 + q_1 A_t) - q_3 A_t, \tag{28}$$

where $\Omega_i = d\phi_i/dt$ is an angular velocity of an ith particle, given by (20), with restricted values (21).

3.2 The maximum energy of the ionized particle

To maximize the energy of the ionized particle we choose the particle 1 to be neutral, i.e. $q_1 = 0$. We are also free to choose the energy of the particle 1, which we set to its rest mass energy, i.e. $E_1 = m_1$ or $\mathcal{E}_1 = 1$. In this case, the angular velocity (20) for the particle 1 will take the following simple form

$$\Omega_1 = \frac{1}{r^2} \sqrt{2(r-2)}. \tag{29}$$

We choose the ionized particle to be the particle 3. The energy of the ionized particle is maximal, when the term $(\Omega_1 - \Omega_2)/(\Omega_3 - \Omega_2)$ is maximized. This occurs when we set the angular momentum of fragments to their limiting values. Then we find

$$\left.\frac{\Omega_1 - \Omega_2}{\Omega_3 - \Omega_2}\right|_{\max} = \frac{1}{\sqrt{2}\, r_{\rm ion}} + \frac{1}{2}. \tag{30}$$

We see that the ratio (30) is maximal, where the ionization point $r_{\rm ion}$ coincides with the horizon of the black hole. Thus, at $r_{\rm ion} = 2$, the ratio (30) is equal to unity. Finally, we write the expression for the energy of the ionized particle in the form

$$E_3 = \left(\frac{1}{\sqrt{2}\, r_{\rm ion}} + \frac{1}{2}\right) E_1 + \frac{q_3 Q}{r_{\rm ion}}. \tag{31}$$

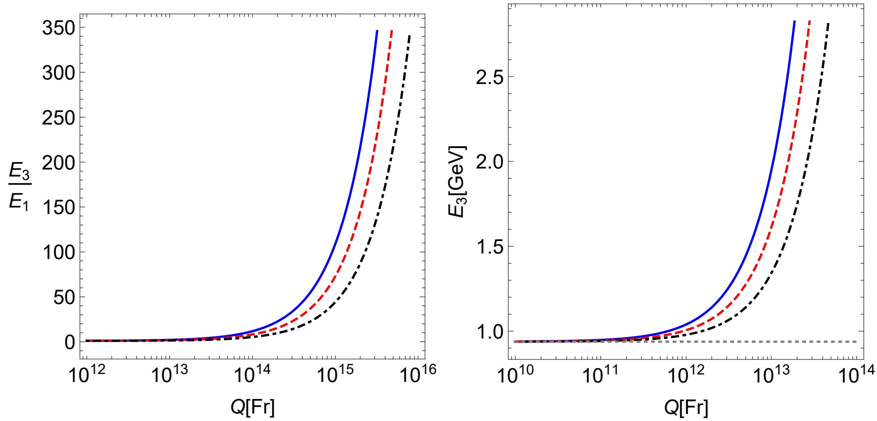

Figure 3. The ratio of energies of escaping proton and the neutral hydrogen and the energy of escaping proton in GeV (left) scaled by factor 100 (right) plotted against the black hole charge Q for the black hole mass $M = 10M_\odot$. The colors of curves denote different positions of the ionization point; blue: $r_{\rm ion} = 2GM/c^2$, red dashed: $r_{\rm ion} = 3GM/c^2$ and black dot-dashed: $r_{\rm ion} = 5GM/c^2$. The dotted grey line corresponds to the rest energy of the proton.

It is useful to define the ratio between the energies of ionized and neutral particles, which would represent the efficiency of the acceleration process. Writing the black hole mass and the speed of light explicitly, we find

$$\frac{E_3}{E_1} = \frac{1}{2} + \sqrt{\frac{M}{2\, r_{\rm ion}}} + \frac{q_3 Q}{m_1 c^2 r_{\rm ion}}. \tag{32}$$

One can see that the energy of the ionized particle is increasing only when q_3 and Q have the same sign, which is also expected.

In order to estimate the process quantitatively, let us consider the ionization of a neutral hydrogen atom consisting of a single proton and single electron in the vicinity of a stellar mass black hole of mass $10M_\odot$. When the electron is separated from the atom, the attained energy of the proton will depend on the charge of the black hole. The proton energy will be larger than the energy of the initially neutral atom, when the charge of the black hole is positive and decrease with increasing the distance between the ionization point and the black hole. We plot the energy of proton after ionization of neutral hydrogen atom in Fig.3, where we also give the ratio of energies of the proton and the neutral hydrogen. We clearly see that the energy of the proton can increase more than 100 times already for the black hole charge values above 10^{15}Fr, which is still more than 15 orders of magnitude smaller charge than the maximal Reissner-Nordström charge (2). Therefore, we conclude that a weakly charged Schwarzschild black hole can act as a high-energy particle accelerator.

4 CONCLUSIONS

We have studied the particle motion and ionization in the vicinity of a non-rotating Schwarzschild black hole carrying the small electric charge, whose gravitational effect on the spacetime metric is negligible. We started from the description of the motion of the charged particle and showed that the effective potential in the case of a weakly charged black hole increases (decreases) with increasing (decreasing) the electric interaction parameter $\mathcal{Q}$. We found that the innermost stable circular orbit (ISCO) of the charged particle increases for both positive and negative values of the parameter $\mathcal{Q}$. The results are in accord with previous similar studies by Pugliese et al. (2011); Zajacek and Tursunov (2019); Hackstein and Hackmann (2020).

We have found that the energy of fthe ionized particle can be much greater than the initial energy of the neutral particle if both charges of the ionized particle and the black hole have the same sign. Thus, the similar acceleration process occurring in the magnetized Kerr black hole spacetime and studied by Tursunov et al. (2020a) works also in the weakly charged non-rotating black hole case. We also estimated the energy of a proton after the ionization of the hydrogen atom in the vicinity of the stellar mass black hole. In this particular situation, we demonstrated that the energy of the proton can increase more than 100 times with respect to the energy of neutral hydrogen when the charge of the black hole is greater than 10^{15}Fr, which is still about 16 orders of magnitude smaller charge than the maximal Reissner-Nordström limit.

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