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Can conformal gravity mimic the rotation of Kerr black hole in terms of particle dynamics?

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ABSTRACT
Testing different theories of gravity through test particle motion around black holes can help deeply understand the nature of the gravity. In this paper we investigated harmonic oscillations of charged particle around a black hole with conformal parameters assuming that a black hole is immersed in the uniform external magnetic field and showed that the increase of the conformal parameters increases the radial frequency ν_r and decreases the other two, ν_ϕ and ν_θ. Then, we considered test particle to be neutral and studied the possibility of mimicking the rotation parameter of Kerr black hole with parameters of black hole in conformal gravity using the results on radius of innermost stable circular orbits (ISCO). We have shown that the conformal parameter L can mimic the spin parameter of Kerr black hole up to $a = 0.45M$ in the case of the parameter $N = 3$ and this value goes down for the smaller values of the parameter N.

Keywords: Conformal gravity – Harmonic oscillations –Kerr black hole – test particle – ISCO.

1 INTRODUCTION

One of the fundamental problems of general theory of relativity is the presence of singularity in almost all known exact analytical solutions of the field equations. For the black hole solutions the central physical singularity with the infinite curvature is unavoidable. There are several attempts to avoid the singularity: coupling with nonlinear electrodynamics (Bardeen, 1968; Hayward, 2006; Ayón-Beato and García, 1998), conformal transformations (Englert et al., 1976; Narlikar and Kembhavi, 1977; Mannheim, 2012; Bars et al., 2014; Bambi et al., 2017, 2016) etc.

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One of the possible ways of excluding the physical singularity in the black hole solutions is using the conformal gravity where metric tensor is transformed as

$$g_{\mu\nu} \to g^{*}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \tag{1}$$

with $\Omega = \Omega(x)$ being a conformal factor of transformation.

Using the modification of Einstein's gravity by the auxiliary scalar field ϕ (dilaton) one may obtain the following Lagrangian for gravity

$$\mathcal{L}_1 = \phi^2 R + 6\, g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)\,. \tag{2}$$

Other efficient way of introducing conformal gravity without introducing dilaton can be performed via following Lagrangian

$$\mathcal{L}_2 = a\, C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} + b\, {}_{*}R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}\,. \tag{3}$$

where $C^{\mu\nu\rho\sigma}$ is the Weyl tensor, $R^{\mu\nu\rho\sigma}$ is the Riemann tensor, ${}_{*}R^{\mu\nu\rho\sigma}$ is the dual of the Riemann tensor, a and b are constants.

In Einstein's theory of gravity the singularity can be resolved by suitable conformal transformation if a spacetime metric $g_{\mu\nu}$ is singular in a gauge. Singularity-free black hole solutions in conformal gravity have been proposed in Refs. (Bambi et al., 2017, 2016). It was shown that these spacetimes are geodetically complete because no massless or massive particles can reach the center of the black hole in a finite amount of time or for a finite value of the affine parameter (Bambi et al., 2017, 2016). Within this theory the curvature invariants do not diverge at the center $r = 0$.

The space-time metric of the spherically symmetric static black hole in Schwarzschild coordinates (t, r, θ, ϕ) in conformal gravity can be described as (Bambi et al., 2017, 2016)

$$ds^2 = S(r)\left[-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2 d\phi^2\right)\right], \tag{4}$$

where $f(r) = f = 1 - 2M/r$ is the lapse function and the scaling factor $S(r)$ has the following form

$$S(r) = S = \left(1 + \frac{L^2}{r^2}\right)^{2N}, \tag{5}$$

with N being a quantity describing conformal gravity assumed to be an integer, L is a new conformal parameter of the black hole coming from the theory.

The electromagnetic fields of slowly rotating neutron stars in conformal gravity have been studied in Ref. Turimov et al. (2018). The authors of Ref. (Zhou et al., 2018) have tested the conformal gravity with the SMBH observation. The energy conditions for conformal gravity are studied in (Toshmatov et al., 2017a) while scalar perturbations of nonsingular nonrotating black holes in conformal gravity have been studied in (Toshmatov et al., 2017b). Charged and magnetized particle motion around rotating non-singular black hole immersed in the external uniform magnetic field in conformal gravity has been studied in

(Narzilloev et al., 2020a; Haydarov et al., 2020) as well as in the spacetime of the quasi-Kerr compact object (Narzilloev et al., 2019). Particle dynamics around the deformed NUT spacetime has been investigated in (Narzilloev et al., 2020).

In most astrophysical observations and measurements of energetic and optical processes around supermassive black hole (SMBH) such as QPOs, ISCO radius measurements (Stuchlík et al. (2020); Kolos, Martin et al. (2020)) and the black hole shadow central gravitating objects are considered as rotating Kerr ones. However, parameters of some alternative and extended theories of gravity may provide similar effects on the processes around the black holes as the spin parameter of the Kerr model. It is one of the problems in relativistic astrophysics where difficulty of distinguishing the central static black hole with the parameters of alternative theories from rotating Kerr black hole takes a place. In our previous papers we have investigated how black hole charge (Rayimbaev et al. (2020); Vrba et al. (2020); Turimov et al. (2020); Narzilloev et al. (2020b)) and different parameters of alternative theories of gravity (Haydarov et al. (2020); Haydarov et al. (2020); Abdujabbarov et al. (2020); Rayimbaev et al. (2020)) can mimic the spin of rotating Kerr black holes proving the same values of ISCO radius for magnetized particles.

No-hair theorem states that black hole can not have its own magnetic moment. However, it is possible to consider a black hole in an external magnetic field generated by external sources. The first approach to get solution of Maxwell equation for the components of the external asypmtotically uniform magnetic field in curved spacetime is Wald's method (Wald (1974)) and during the past years the method has been developed by several authors (Aliev et al. (1986); Aliev and Gal'tsov (1989); Aliev and Özdemir (2002); Stuchlík et al. (2014); Stuchlík and Kološ (2016)).

This work is devoted to the study of test particle motion around non rotating compact object in conformal gravity and organized as follows: The Sect. 2 is devoted to the study of charged particle motion where we mostly focus on the QPOs of charged particles. In Sect. 3 neutral particles motion around black hole in conformal gravity have been investigated where we discuss possible ways of mimicking the rotation parameter of Kerr black hole with confomal parameters. We summarize our results in Sect. 5.

Throughout this work we use signature $(-,+,+,+)$ for the spacetime and geometrized unit system $G = c = 1$ (However, for an astrophysical application we have written the speed of light and Newtonian constant explicitly in our expressions). Latin indices run from 1 to 3 and Greek ones from 0 to 3.

2 CHARGED PARTICLE MOTION

In this section we study charged particle motion around a black hole in conformal gravity in the presence of the external uniform magnetic field. The Hamilton-Jacobi equation for test particle with mass m and the charge e can be expressed as (Narzilloev et al., 2020b)

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^{\mu}} + eA_{\mu}\right)\left(\frac{\partial S}{\partial x^{\nu}} + eA_{\nu}\right) = -m^2 \, , \tag{6}$$

where $A^\alpha = (0, 0, 0, B/2)$ is a four vector potential of the external magnetic field B. The solution of equation (6) can be sought in the following form

$$\mathcal{S} = -\mathcal{E}t + \mathcal{L}\phi + \mathcal{S}_r(r) + \mathcal{S}_\theta(\theta) , \tag{7}$$

where $\mathcal{E}$ and $\mathcal{L}$ are the energy and the angular momentum of the test particle respectively.

It is convenient to consider particle motion on a constant plane $\dot{\theta} = 0$ ($p_\theta = 0$) and one can write the radial part as

$$\dot{r}^2 + V_{\text{eff}}(r; \theta) = \mathcal{E}^2 , \tag{8}$$

where the effective potential has a form

$$V_{\text{eff}}(r; \theta) = f(r)\left[S(r) + \left(\frac{\mathcal{L}}{r\sin\theta} + \omega_{\text{B}} r S(r)\sin\theta\right)^2\right] , \tag{9}$$

with magnetic coupling parameter $\omega_{\text{B}} = eB/(2mc)$ or so-called cyclotron frequency which characterizes the interaction between charged particle and the external magnetic field.

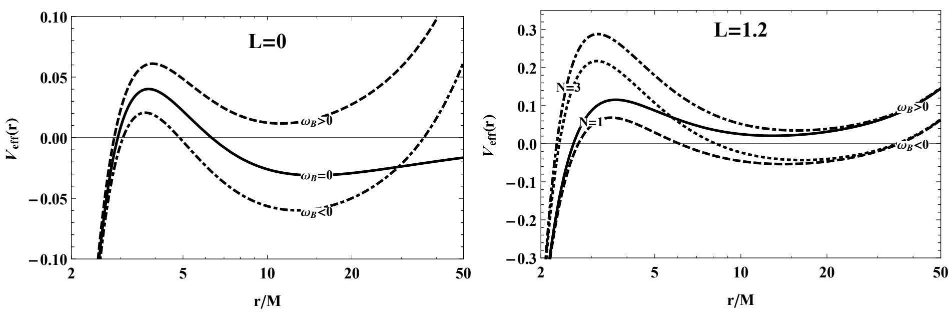

Figure 1. The radial dependence of effective potential of charged particle for the different values of the conformal, scale, and magnetic coupling parameters.

Fig. 1 shows the radial dependence of the effective potential of the charged particle. One can see from the figure that when $\omega_{\text{B}} > 0$ effective potential is bigger than the case of $\omega_{\text{B}} < 0$ and it increases with the increase of the values of parameters L and N. It is worth to note that at large distances the effect of magnetic field plays an important role rather than the effect of conformal gravity.

2.1 Harmonic oscillations

If a charged test particle is slightly displaced from the equilibrium position, at r_0 and $\theta_0 = \pi/2$, being stable circular orbit, which corresponds to the minimum of the effective potential $V_{\text{eff}}(r, \theta)$ the particle will start oscillating around the minimum realizing thus epicyclic motion governed by linear harmonic oscillations. For harmonic oscillations around the minimum of the effective potential $V_{\text{eff}}(r)$, the evolution of the displacement of

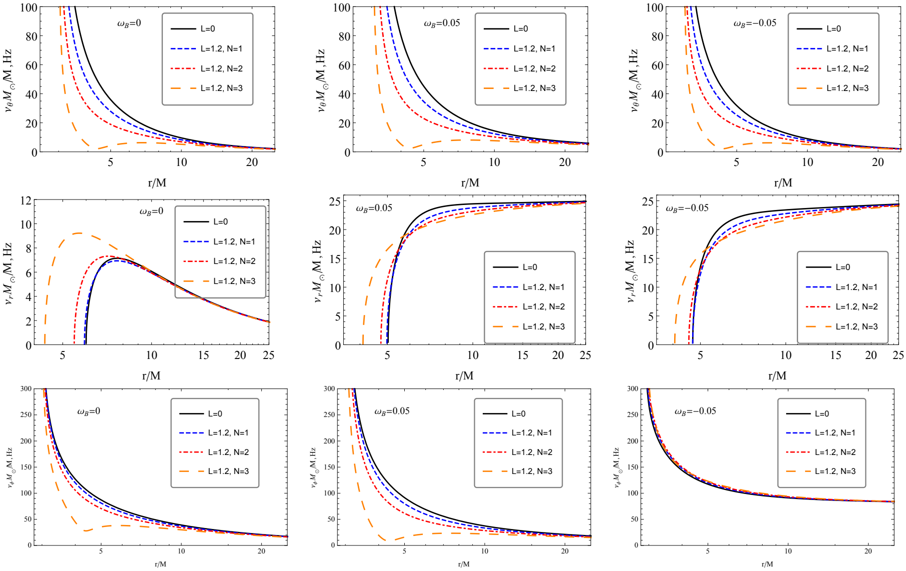

Figure 2. Radial profiles of fundamental frequencies of charged particles around Schwarzschild black hole in conformal gravity, measured by an observer at infinity, for different values of magnetic coupling $\omega_{\rm B}$ and conformal parameters L and N .

coordinates reads $r = r_0 + \delta r$, $\theta = \theta_0 + \delta\theta$. Locally measured angular frequencies of the harmonic oscillators can be expressed as

$$\omega_r^2 = \frac{1}{g_{rr}}\frac{\partial^2 V_{\rm eff}}{\partial r^2} , \tag{10}$$

$$\omega_\theta^2 = \frac{1}{g_{\theta\theta}}\frac{\partial^2 V_{\rm eff}}{\partial \theta^2} , \tag{11}$$

$$\omega_\phi = \mathcal{L} - g_{\phi\phi}\omega_{\rm B} . \tag{12}$$

Frequencies themselves can be written using the following expression in the unit of Hz

$$\nu_i = \frac{1}{2\pi}\frac{c^3}{GM}\Omega_i , \tag{13}$$

where $i = r, \theta, \phi$ and

$$\Omega_i = \frac{1}{\sqrt{-g_{tt}}}\omega_i . \tag{14}$$

The radial dependence of fundamental frequencies of particles is presented in Fig. 2. One can see from the figures that values of radial oscillations increase with the increase of L,N while others decrease with the increase of latter.

3 TEST PARTICLE MOTION AROUND BLACK HOLE IN CONFORMAL GRAVITY

In this part we restrict our calculations considering the test particle to be electrically neutral and investigate its motion around static black hole in conformal gravity. The Hamilton-Jacobi equation of motion of test particles (6) reduces to

$$g^{\mu\nu}\frac{\partial \mathcal{S}}{\partial x^\mu}\frac{\partial \mathcal{S}}{\partial x^\nu} = -m^2 \, . \tag{15}$$

On the equatorial plane ($\theta = \pi/2$) the equations of motion can be expressed using conservative quantities, specific energy $\mathcal{E}$ and angular momentum l as

$$\dot{t} = \frac{\mathcal{E}}{f\,\mathrm{S}} \, , \tag{16}$$

$$\dot{r}^2 = \mathcal{E}^2 - f\mathrm{S}\left(1 + \frac{l^2}{r^2}\right) , \tag{17}$$

$$\dot{\phi} = \frac{l}{\mathrm{S}r^2} \, . \tag{18}$$

One can define the effective potential of radial motion of magnetized particles on equatorial plane as

$$\dot{r}^2 = \mathcal{E}^2 - 1 - 2V_{\rm eff} \, , \tag{19}$$

where the effective potential has the following form

$$V_{\rm eff} = \frac{1}{2}\left[f\,\mathrm{S}\left(1 + \frac{l^2}{r^2}\right) - 1\right] , \tag{20}$$

Now we will consider orbits of test particles to be circular, or more specifically the innermost stable ones. Using the following standard conditions

$$V_{\rm eff}(r) = \mathcal{E}^2 \, , \qquad V'_{\rm eff}(r) = 0 \, , \qquad V''_{\rm eff}(r) = 0 \, , \tag{21}$$

one can easily find the values of ISCO radius. Angular momentum for circular orbits can also be found from the equations above that reads

$$\mathcal{L} = \frac{r^2\left[L^2(4MN + M - 2Nr) + Mr^2\right]}{L^2[r(2N+1) - M(4N+3)] + r^2(r - 3M)} \, . \tag{22}$$

The energy of the charged particle at circular orbits will have the following form

$$\mathcal{E} = \frac{\left(L^2 + r^2\right)(r - 2M)^2\left(1 + \frac{L^2}{r^2}\right)^{2N}}{r\left\{L^2[r(2N+1) - M(4N+3)] + r^2(r - 3M)\right\}} \, . \tag{23}$$

Fig. 3 illustrates the radial profiles of the angular momentum and energy of the test particle at circular orbits on equatorial plane. One can see that the angular momentum and

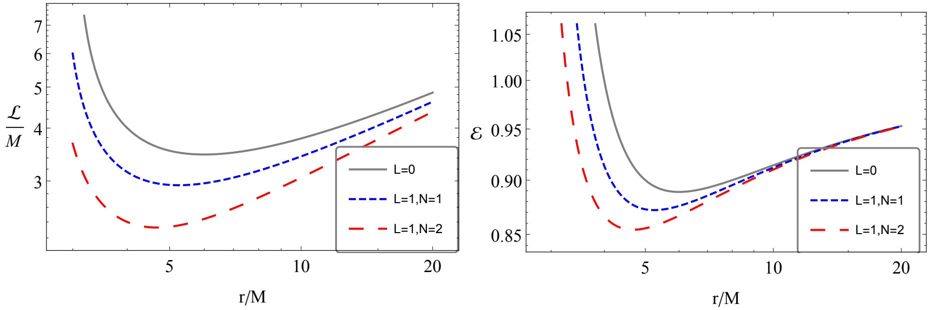

Figure 3. Redial dependence of specific angular momentum ($\mathcal{L}$ at the left panel) and energy ($\mathcal{E}$ at the right panel) of the particles for circular stable orbits

the energy of the particle decrease in the presence of conformal parameters. It can be also seen that the plots are shifted to the left which corresponds to the decrease of the ISCO.

Using the condition for the stability of circular orbits ($\partial_r^2 V_{\rm eff} \geq 0$) one can write

$$L^4\left[M(8N(4N+3)+1)r-2M^2(4N+1)(4N+3)-4N(2N+1)r^2\right] \\ +2L^2Mr^2[r(2N+1)-M(4N+3)]+Mr^4(r-6M)\geq 0\,. \qquad (24)$$

One can see from Eq. (24) that in the absence of conformal part $L = 0$ we get $r_{\rm isco} = 6M$ being the value of ISCO in Schwarzschild case. The analytical form of the solution of Eq. (24) is quite complicated and it would be difficult to see the effects of conformal parameters L and N on ISCO radius of the test particle from such expression. Thus, it is better to show detailed analysis in plot form which we will do here.

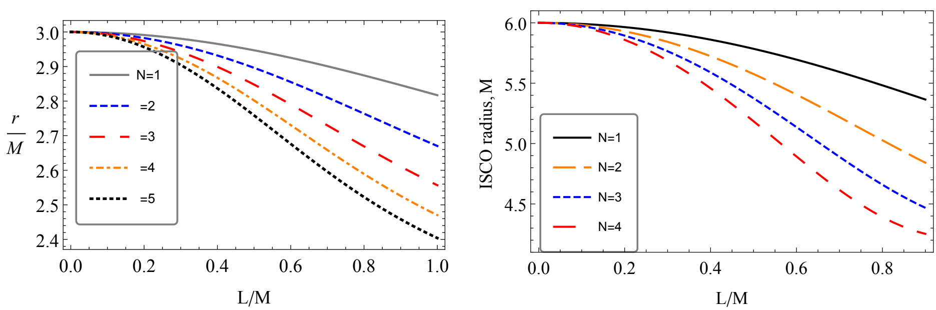

Figure 4. Dependence of the minimal distance of circular orbits (left panel) and ISCO radius (right panel) of test particles around black hole in conformal gravity on the conformal parameter L for the fixed values of the parameter N.

From the condition above one can get the relation between the ISCO radius and conformal parameters. The Fig. 4 shows the profiles of ISCO radius depending on conformal parameters. One can see that the increase of both conformal parameters causes to decrease the ISCO radius and the minimal radius of circular orbits. One can also mention that both plots have similar shapes for the given ranges of conformal parameter L.

3.1 Can conformal gravity parameters mimic the rotation parameter of Kerr BH?

As an astrophysical application of the studies of the ISCO of neutral particles orbiting around the non-rotating black hole in conformal gravity, we consider here the possibility of mimicking the spin parameter a of Kerr black hole with the parameters L and N of black hole in conformal gravity using the results for ISCO.

The ISCO radius of the test particles for co-rotating orbits around Kerr BH is given by the relation (Bardeen et al., 1972)

$$r_{\rm ISCO} = 3 + Z_2 - \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}\,, \tag{25}$$

where

$$Z_1 = 1 + \left(\sqrt[3]{1-a} + \sqrt[3]{1+a}\right)\sqrt[3]{1-a^2}\,, \qquad Z_2 = \sqrt{3a^2 + Z_1^2}\,.$$

But, since in the given form the value of ISCO radius depends on the choice of coordinate system we need to deal with invariant quantity that defines the ISCO radius to compare the results in two different spacetime metrics. As for such invariant quantity we use the line element which takes the following form on equatorial plane where we set all the coordinates to constants except the coordinate ϕ

$$ds_\phi = \sqrt{g_{\phi\phi}}d\phi\,, \tag{26}$$

and after integrating the length of such circular orbit becomes

$$l_{ISCO} = 2\pi\sqrt{g_{\phi\phi}}|_{r=r_{ISCO}}\,. \tag{27}$$

The invariant ISCO radius then can be defined as

$$R_{ISCO} = \frac{l_{ISCO}}{2\pi}\,, \tag{28}$$

and one can get the degeneracy between the spin of the Kerr metric and conformal parameters for the matching value of such radius obtained for these two spacetime metrics.

Now we may investigate how well the conformal parameters can mimic the rotation parameter of Kerr one through the matching invariant ISCO radius, R_{ISCO}. In Fig. 5 we show the degeneracy between rotation parameter of Kerr black hole and static black hole with conformal parameters. One may see that the conformal parameter L can mimic the spin parameter of Kerr black hole providing the same value for ISCO radius of test particles up to the value of $a/M \simeq 0.45$ when $N = 3$ and such mimicking value becomes smaller with the decrease of the parameter N and it takes the value $a/M \simeq 0.3$ for $N = 1$.

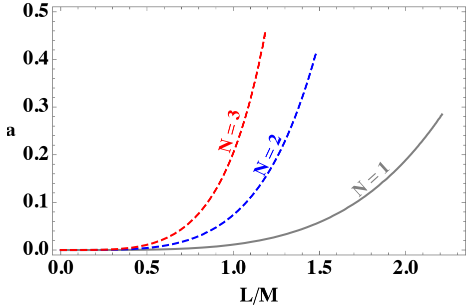

Figure 5. Relation that shows how the spin parameter a of Kerr black hole and corresponding conformal parameter L provide the same ISCO radius for test particles for the different values of the parameter N.

4 CONCLUSION

In this work, we have studied charged particle motion around a static black hole with conformal paramters immersed in the external uniform magnetic field. Investigation of the QPO for the charged particle showed that the frequency of the radial oscillation becomes higher for bigger conformal parameters while the other fundamental frequencies have opposite behaviour. We have also studied circular motion of neutral test particles around Schwarzschild black hole in conformal gravity. Analysis of the studies of specific energy and angular momentum for circular stable orbits show that the increase of both conformal parameters cause to decrease of the energy and angular momentum. It is obtained that the value of ISCO radius decreases with the increase of both conformal parameters. Comparisons of the ISCO of the test particle around a black hole in conformal gravity with the Kerr black hole show that the conformal parameter L can mimic the spin of Kerr black hole up to $a = 0.45$ in the case of the parameter $N = 3$ and this value decreases with the decrease of the parameter N.

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