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Dynamics of magnetized particles around Reissner-Nordström black holes

Javlon Rayimbaev[1,2,3,a] and Nozima Juraeva[1,b]

[1] Ulugh Beg Astronomical Institute, Astronomicheskaya 33, Tashkent 100052, Uzbekistan,
[2] National University of Uzbekistan, Tashkent 100174, Uzbekistan
[3] Institute of Nuclear Physics, Ulughbek, Tashkent 100214, Uzbekistan
[a] javlon@astrin.uz
[b] nozima@astrin.uz

ABSTRACT
This paper is devoted to studying the dynamics of magnetized particles around electrically charged Reissner-Nordström (RN) black hole immersed in an external asymptotically uniform magnetic field. Here, we have focused on the effects of the external magnetic field and the electric charge of the RN black hole on the range of stable circular orbits for magnetized particles. We have shown that the dimensionless magnetic interaction parameter between magnetic dipole moment of a magnetized particle orbiting the black hole and the external magnetic field must be less than 1 in absolute value in order to allow stable circular orbits and with increasing of the electric charge of the black hole the range where circular orbits are allowed decreases while the increase of external magnetic field causes to increase it.

Keywords: Magnetized particles –charged black hole –external magnetic field

1 INTRODUCTION

Particle motion around a compact gravitational object is the special subject of highly motivated interest, since it may be used to develop new tests and probe the theories describing the gravitational interaction in the strong field regime. Particularly, black hole as gravitational compact object is useful astrophysical object to study the particle dynamics around the latter. Being simple object astrophysical black holes can be described with a few parameters, namely black hole total mass M, rotation parameter a, and electric charge Q. Static spherically-symmetric electrically charged black hole is described by the Reissner-Nordström (RN) solution Reissner (1916); Nordström (1918).

The electromagnetic field surrounding black hole or neutron star plays an important role in astronomical observation of compact objects through electromagnetic radiation or its influence to astrophysical processes around it. Even in the case of test electromagnetic field, when electromagnetic potential does not change the spacetime structure, the influence of the electromagnetic field to the energetic and dynamical processes around gravitational compact objects is essential. Despite no hair theorem, according to which the black

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hole cannot have its own intrinsic magnetic field Misner et al. (1973) one may explore the external magnetic field surrounding the black hole. Particularly the electromagnetic field structure around rotating black hole embedded in an external asymptotically uniform magnetic field has been studied in the pioneering paper Wald (1974). In past years, different properties of the electromagnetic fields in the vicinity of black holes immersed in external asymptotically uniform magnetic fields and proper magnetic field of rotating magnetized neutron stars with dipolar structure were widely studied by several authors in different models of gravity Aliev et al. (1986); Aliev and Gal'tsov (1989); Aliev and Özdemir (2002); Benavides-Gallego et al. (2019); Stuchlík et al. (2014); Stuchlík and Kološ (2016); Rayimbaev and Tadjimuratov (2020); Rayimbaev et al. (2015, 2019b,a, 2020). This electromagnetic field will change the dynamics of charged particle in close black hole environment Chen et al. (2016); Hashimoto and Tanahashi (2017); Dalui et al. (2019); Han (2008); de Moura and Letelier (2000); Morozova et al. (2014); Narzilloev et al. (2020). Together with charged particles one may study the influence of electromagnetic field near the black hole to magnetized particle motion. The dynamics of particles with intrinsic nonzero dipolar magnetic field around non-rotating and rotating black holes immersed in external magnetic field have been studied in de Felice and Sorge (2003); de Felice et al. (2004). Our recent works have been devoted to study the magnetized particle motion around black hole in magnetic field in different gravity models and theories Rayimbaev (2016); Rayimbaev et al. (2020); Toshmatov et al. (2015); Abdujabbarov et al. (2014); Rahimov et al. (2011); Rahimov (2011); Haydarov et al. (2020); Haydarov et al. (2020); Abdujabbarov et al. (2020); Vrba et al. (2020).

The paper is organized as follows: In Sect. 2 we study dynamics of magnetized particles around electrically RN black hole immersed in an external asymptotically uniform magnetic field in comoving observer frame. Finally, we summarize our results in Sect. 3.

We use the space-time signature $(-,+,+,+)$ and geometrized units system $G_{\mathrm{N}} = c = 1$. The Latin indices are expected to run from 1 to 3 and the Greek ones from 0 to 3.

2 MAGNETIZED PARTICLE MOTION AROUND ELECTRICALLY CHARGED RN BLACK HOLE IN MAGNETIC FIELD

The spacetime exterior to electrically charged RN black hole with total mass M and electric charge Q can be described by the metric:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \, , \tag{1}$$

where the radial metric function is

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \, . \tag{2}$$

Since there is no interaction between magnetized particles and electric charged RN black hole we assume that the black hole immersed in an external asymptotically uniform magnetic field and finally, the electromagnetic four-potentials can be expressed using the Wald method Wald (1974) in the following form

$$A_\phi = \frac{1}{2} B_0 r^2 \sin^2\theta \,, \tag{3}$$

$$A_t = -\frac{Q}{r} \,, \tag{4}$$

where B_0 is asymptotic value of the external uniform magnetic field. One may immediately find the non-zero components of the electromagnetic tensor using the definition $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ in the following form

$$F_{r\phi} = B_0 r \sin^2\theta \,, \tag{5}$$

$$F_{\theta\phi} = B_0 r^2 \sin\theta \cos\theta \,, \tag{6}$$

$$F_{rt} = \frac{Q}{r^2} \tag{7}$$

The orthonormal components of the magnetic field around electrically charged RN black hole measured by proper observer

$$B^\alpha = \frac{1}{2} \eta^{\alpha\beta\sigma\mu} F_{\beta\sigma} w_\mu \,, \tag{8}$$

where w_μ is four-velocity of the proper observer, $\eta_{\alpha\beta\sigma\gamma}$ is the pseudo-tensorial form of the Levi-Civita symbol $\epsilon_{\alpha\beta\sigma\gamma}$ with the relations

$$\eta_{\alpha\beta\sigma\gamma} = \sqrt{-g}\epsilon_{\alpha\beta\sigma\gamma} \qquad \eta^{\alpha\beta\sigma\gamma} = -\frac{1}{\sqrt{-g}} \epsilon^{\alpha\beta\sigma\gamma} \,, \tag{9}$$

and $g = \det|g_{\mu\nu}| = -r^4 \sin^2\theta$ for spacetime metric (1) are

$$B^{\hat r} = B_0 \cos\theta, \qquad B^{\hat\theta} = \sqrt{f(r)} B_0 \sin\theta \,. \tag{10}$$

According to de Felice and Sorge (2003) the equation of motion of magnetized particles in the spacetime of a black hole immersed in the external magnetic field can be described by the following Hamilton-Jacobi equation

$$g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = -\left(m - \frac{1}{2} D^{\mu\nu} F_{\mu\nu} \right)^2 \,, \tag{11}$$

where m is mass of the particle, S is the action for magnetized particles in the spacetime of the black hole, the scalar term came from the product of polarization and electromagnetic field tensors $D^{\mu\nu}F_{\mu\nu}$ being responsible for the interaction between the external magnetic field and dipole moment of magnetized particles. The polarization tensor $D^{\mu\nu}$ corresponding to the magnetic dipole moment of magnetized particles is describes by the relation de Felice and Sorge (2003):

$$D^{\alpha\beta} = \eta^{\alpha\beta\sigma\nu} u_\sigma \mu_\nu, \qquad D^{\alpha\beta} u_\beta = 0 \,, \tag{12}$$

where μ^ν and u^ν are the four-vector of magnetic dipole moment and four-velocity of magnetized particles by the fiducial comoving observer. The electromagnetic field tensor can be decomposed through $F_{\alpha\beta}$ by electric E_α and magnetic B^α field components as

$$F_{\alpha\beta} = u_{[\alpha}E_{\beta]} - \eta_{\alpha\beta\sigma\gamma}u^\sigma B^\gamma . \tag{13}$$

One can find the product of polarization and electromagnetic tensors taking account the condition given in Eq. (12) in the following form

$$D^{\mu\nu}F_{\mu\nu} = 2\mu^{\hat{\alpha}}B_{\hat{\alpha}} = 2\mu B_0 \mathcal{L}[\lambda_{\hat{\alpha}}] , \tag{14}$$

where $\mu = \sqrt{|\mu_{\hat{i}}\mu^{\hat{i}}|}$ is the norm of the dipole magnetic moment of magnetized particles and $L[\lambda_{\hat{\alpha}}]$ is the tetrad $\lambda_{\hat{\alpha}}$ attached to the comoving fiducial observer being the function of the radial coordinate and the black hole parameters.

Here we investigate dynamics of magnetized particles in circular orbits around electrically charged RN black hole in the weak magnetic interaction approximation due to weakness of the external magnetic field or/and the particle is less magnetized

$$\left(D^{\mu\nu}F_{\mu\nu}\right)^2 \to 0.$$

This approximation is astrophysical relevant since the external magnetic fieldin black hole environment is comparatively weak (Piotrovich et al. (2011)). The action for magnetized particles at the equatorial plane (where $\theta = \pi/2$ and $\dot{\theta} = 0$) can be described by the following form

$$\mathcal{S} = -Et + L\phi + \mathcal{S}_r , \tag{15}$$

which allows to separate the variables in Hamilton-Jacobi equation. The equation of radial motion of magnetized particles can be found as

$$\dot{r}^2 = \mathcal{E}^2 - V_{\rm eff}(r, Q, l, b) , \tag{16}$$

where the effective potential of radial motion of magnetized particles has the following form:

$$V_{\rm eff}(r, Q, l, b) = f\left(1 + \frac{l^2}{r^2} - b\mathcal{L}[\lambda_{\hat{\alpha}}]\right) , \tag{17}$$

where $b = 2\mu B_0/m$ is magnetic interaction parameter responsible to the interaction between dipole moment of magnetized particles and external magnetic field and $l = L/m$ is specific angular momentum of magnetized particles.

In real astrophysical scenarios one may treat a neutron star with the magnetic dipole moment $\mu = (1/2)B_{\rm NS}R^3_{\rm NS}$, like a magnetized particle, orbiting a supermassive black hole (SMBH) immersed in an external magnetic field with different configurations. In such a case the magnetic coupling parameter b can easily be estimated through the observational parameters of the neutron star and the approximate value of the external magnetic field around the SMBH in the following form:

$$b = \frac{B_{\rm NS}R^3_{\rm NS}B_{\rm ext}}{m_{\rm NS}} \simeq \frac{\pi}{10^3}\left(\frac{B_{\rm NS}}{10^{12}{\rm G}}\right)\left(\frac{B_{\rm ext}}{10{\rm G}}\right)\left(\frac{R_{\rm NS}}{10^6{\rm cm}}\right)^3\left(\frac{m_{\rm NS}}{1.4M_\odot}\right)^{-1} . \tag{18}$$

One may apply the calculation to estimate the value of the magnetic coupling parameter for a realistic case of the magnetar SGR (PSR) J1745–2900 orbiting around Sagittarius A* (Sgr A*). In the estimation of the coupling parameter we have considered magnetic field around SgrA* is in the order of 10 G, the magnetic dipole moment of the magnetar $\mu \approx 1.6 \times 10^{32} \mathrm{G} \cdot \mathrm{cm}^3$ and its mass $m \approx 1.4 M_\odot$ (Mori et al. (2013)) as

$$b_{\mathrm{PSRJ1745-2900}} \simeq 0.716 \left(\frac{B_{\mathrm{ext}}}{10\mathrm{G}}\right) . \tag{19}$$

Generally, the circular motion of test particles around axially symmetric black holes describes by the following standard condition

$$\dot{r} = 0, \qquad \frac{\partial V_{\mathrm{eff}}(r; Q, l, b)}{\partial r} = 0 . \tag{20}$$

The explicit expressions for the orthonormal tetrad carried the fiducial observer $\mathcal{L}[\lambda_{\hat{\alpha}}]$ can be formulated for circular motion at the equatorial plane around spherical symmetric black hole in the following form

$$\lambda_{\hat{t}} = e^{\Psi} \left(\partial_t + \Omega \partial_\phi\right) , \tag{21}$$

$$\lambda_{\hat{r}} = e^{\Psi} \left[-\frac{\Omega r}{\sqrt{f(r)}} \partial_t - \frac{\sqrt{f(r)}}{r} \partial_\phi \right] \sin(\Omega_{FW} t) + \sqrt{f(r)} \cos(\Omega_{FW} t) \partial_r , \tag{22}$$

$$\lambda_{\hat{\theta}} = \frac{1}{r} \partial_\theta , \tag{23}$$

$$\lambda_{\hat{\phi}} = e^{\Psi} \left[\frac{\Omega r}{\sqrt{f(r)}} \partial_t + \frac{\sqrt{f(r)}}{r} \partial_\phi \right] \cos(\Omega_{FW} t) + \sqrt{f(r)} \sin(\Omega_{FW} t) \partial_r , \tag{24}$$

where Ω_{FW} is Fermi-Walker angular velocity de Felice and Sorge (2003) and

$$e^{-\Psi} = \sqrt{f(r) - \Omega^2 r^2} , \tag{25}$$

with Ω is the angular velocity of the particles measured by a distant observer defined as

$$\Omega = \frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}\phi/\mathrm{d}\tau}{\mathrm{d}t/\mathrm{d}\tau} = \frac{f(r)}{r^2} \frac{l}{\mathcal{E}} . \tag{26}$$

We will study the motion of a magnetized particle orbiting at the equatorial plane assuming the magnetic dipole moment of the magnetized particle is always perpendicular to the equatorial plane and parallel to the external magnetic field. The orthonormal components of the external magnetic field measured by the observer comoving with the magnetized particle take the following form

$$B_{\hat{r}} = B_{\hat{\phi}} = 0 , \qquad B_{\hat{\theta}} = B_0 f(r)\, e^{\Psi} . \tag{27}$$

The induced electric field measured by the comoving observer can be written as

$$E_{\hat{r}} = B_0 \Omega r \cos(\Omega_{FW} t) \sqrt{f(r)} e^{\Psi} , \tag{28}$$
$$E_{\hat{\theta}} = 0 , \tag{29}$$
$$E_{\hat{\phi}} = B_0 \Omega r \sin(\Omega_{FW} t) \sqrt{f(r)} e^{\Psi} . \tag{30}$$

In case when the Fermi-Walker and the particles angular velocities measured by proper observer are zero ($\Omega_{FW} = \Omega = 0$), the above tetrad turns to tetrad of the proper observe (see Rezzolla and Zanotti (2001) in the case $a = 0$) and the magnetic field components in Eq.(27) equals to the components in Eq.(10) and the induced electric field vanishes.

One may find the possible values of the magnetic coupling parameter β for circular orbits from the first condition in Eq, (20)

$$b(r; l, \mathcal{E}, Q) = \frac{1}{\mathcal{L}[\lambda_{\hat{\sigma}}]} \left(1 + \frac{l^2}{r^2} - \frac{\mathcal{E}^2}{f(r)} \right) . \tag{31}$$

Inserting Eq.(27) into (14) we can find the interaction part of the Eq. (11)

$$D \cdot F = 2\mu B_0 f(r)\, e^{\Psi} , \tag{32}$$

One can find the analytic form of the term $\mathcal{L}[\lambda_{\hat{\sigma}}]$ by the comparison of Eq.(32) with Eq.(14) in the following form

$$\mathcal{L}[\lambda_{\hat{\sigma}}] = e^{\Psi} f(r) . \tag{33}$$

Finally, the magnetic interaction parameter $b(r; l, \mathcal{E}, Q)$ for stable circular orbits has the following form

$$b(r; l, \mathcal{E}, Q) = \left(\frac{1}{f(r)} - \frac{l^2}{\mathcal{E}^2 r^2} \right)^{1/2} \left(1 + \frac{l^2}{r^2} - \frac{\mathcal{E}^2}{f(r)} \right) . \tag{34}$$

Eq. (34) implies that a magnetized particle with magnetic coupling parameter b corresponds to circular stable orbit r with the energy $\mathcal{E}$ and angular momentum l.

Fig. 1 shows the radial dependence of the magnetic coupling parameter b for the different values of electric charge of RN black hole for the fixed values of the specific energy and angular momentum. One can see from the top panel of Fig. 1 that the maximal (minimal) values of the magnetic interaction parameter for the fixed values of the specific energy $\mathcal{E} = 0.9$ and angular momentum $l/M = 4$ ($l/M = 2\sqrt{5}$) increases (decreases) with the increase of electric charge of RN black hole.

Now we analyze the values of the magnetic interaction parameter corresponding to stable orbits for magnetized particles. It can be found using following set of equations de Felice and Sorge (2003); Rayimbaev (2016):

$$b = b(r; l, \mathcal{E}, Q), \qquad \frac{\partial b(r; l, \mathcal{E}, Q)}{\partial r} = 0 . \tag{35}$$

One can see from Eq. (35) contains two system of equations including five free parameters: four of them related to the magnetized particle $(b, r, l, \mathcal{E})$ and one to the spacetime (Q),

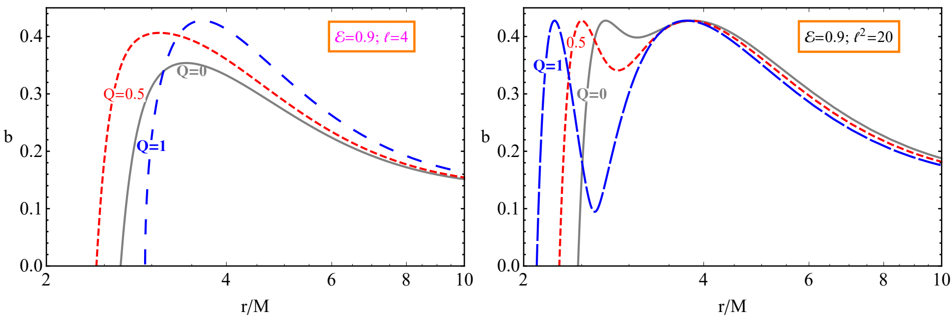

Figure 1. The radial dependence of magnetic coupling parameter for the different values of electric charge of RN black hole. In plots we used the values for the specific energy $\mathcal{E} = 0.9$ and for angular momentum $l = 4M$ (top panel) and $l = 2\sqrt{5}M$ (bottom panel). The black hole charge and specific angular momentum read as $Q \to Q/M$ and $l \to l/M$, respectively.

so its solution can be found in terms of any two of the five parameters as independent variables. To solve the system of equations, we prefer to use the magnetic interaction parameter b and radius of the circular orbits r as free parameters. The specific energy $\mathcal{E}$ and the angular momentum l of the magnetized particle are considered as a functions of the radial coordinate and electric charge of RN black hole Q:

$$\mathcal{E}_{\min}(r; l, Q) = \frac{l\left[r(r-2M)+Q^2\right]}{r^2\sqrt{Mr-Q^2}} . \tag{36}$$

The expression (36) corresponds to the possible values of the specific energy of the magnetized particle at stable circular orbits.

Fig. 2 demonstrates radial dependence of the specific energy of magnetized particles corresponding to circular motion for the different values of electric charge of RN black hole at the fixed value of the specific angular momentum $l = 4M$. One can see from the figure that the energy increases with the increase of the black hole charge due to increase of gravitational potential of the spacetime around RN black hole and inner circular orbit's position shifts towards to the central black hole.

Substituting (36) into (34) one to calculate the minimum value of the magnetic interaction parameter of the magnetic particle for the fixed value of the specific energy as

$$b_{\min}(r; l, Q) = \frac{\sqrt{r(r-3M)+2Q^2}}{r\left(Mr-Q^2\right)\left[r(r-2M)+Q^2\right]} \times \left\{l^2\left(3Mr-2Q^2-r^2\right)+Mr^3-Q^2r^2\right\}. \tag{37}$$

Figure 3 illustrates the radial dependence of the minimal values of magnetic interaction parameter of magnetized particles corresponding to circular orbits at the value of the specific angular momentum $l = 2\sqrt{5}M$ for different values of electric charge of RN black hole.

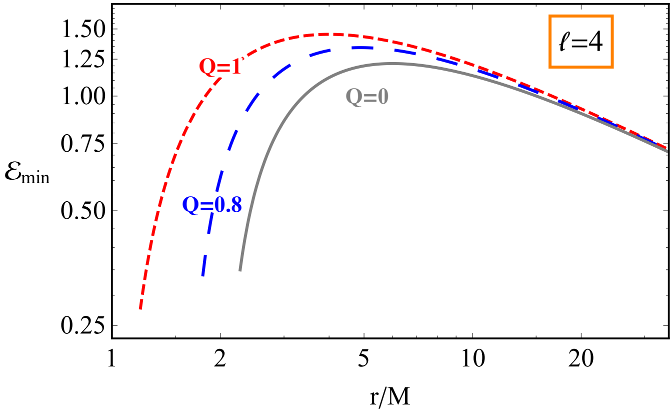

Figure 2. Radial profile of minimal energy of the magnetized particle for the different values of electric charge of RN black hole. Read the specific angular momentum and electric charge of RN black hole as $l \to l/M$ and $Q \to Q/M$, respectively.

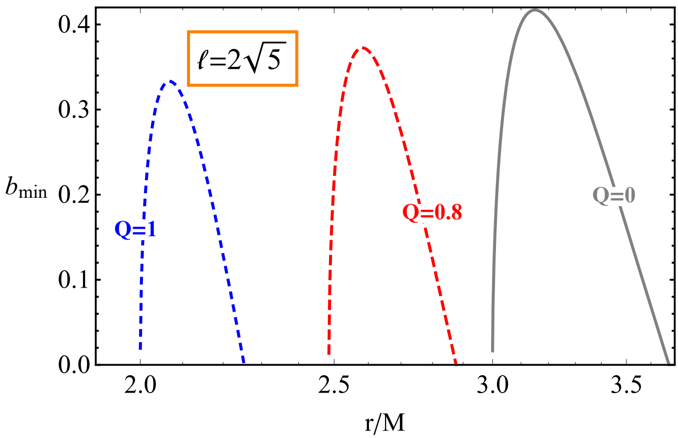

Figure 3. Radial profile of minimal value of magnetic interaction parameter b of magnetized particles with the specific angular momentum $l/M = 2\sqrt{5}$ for the different values of electric charge of RN black black hole. Units of the black hole charge and specific angular momentum are given in the unit of mass M

One can see that the maximal values of minimum interaction parameter decreases with the increase of electric charge of the black hole and the circular orbits corresponding to the fixed specific angular momentum shifts towards the black hole at the center of the orbits.

Let us consider the maximum value for the angular momentum that the particle can be in stable circular orbits which can be found using condition $\partial b_{\min}/\partial r = 0$ as:

$$l_{\min}(r; Q) = \frac{r\left(Q^2 - Mr\right)}{\sqrt{[r(r-3M) + 2Q^2]\,[r(2r-3M) + Q^2]}}. \tag{38}$$

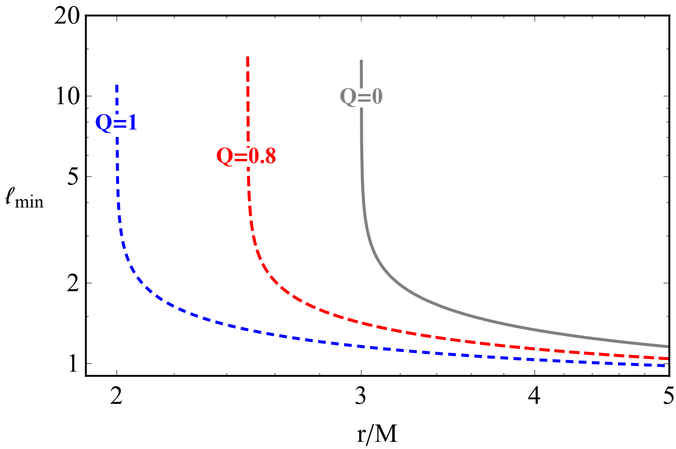

Figure 4. Radial profiles of minimum specific angular momentum of magnetized particles corresponding to stable circular orbits around charged RN black hole for the different values of electric charge of the black hole. Here the unit of the specific angular momentum and the black hole charge are given the unit of mass M.

Figure 4 shows radial dependence of minimum specific angular momentum for stable circular orbits of magnetized particles around RN black hole for the different values of the electric charge of the black hole. One can see that the value of minimum specific angular momentum is increased with increasing electric charge of RN black hole and the position where the angular momentum is maximum also shifts towards central black hole.

The extreme value of the magnetic interaction parameter b can be found by inserting equation (38) into the equation (37) in the following form

$$b_{\text{extr}}(r; Q) = \frac{2r\sqrt{r(r-3M) + 2Q^2}}{r(2r-3M) + Q^2}\ . \tag{39}$$

Figure 5 demonstrates radial dependence of minimum magnetic interaction parameter for freely falling magnetized particles and extreme values of the interaction parameter corresponding to stable circular orbits of magnetized particles around RN black hole for the different values of electric charge of RN black hole. The colored area shows possible value of magnetic interaction parameter corresponding to the area where stable circular orbits are allowed. Dashed lines correspond to the minimum values of the b parameter at $l = 0$ and solid ones correspond to the extreme value of the parameter b. Gray, light-blue and

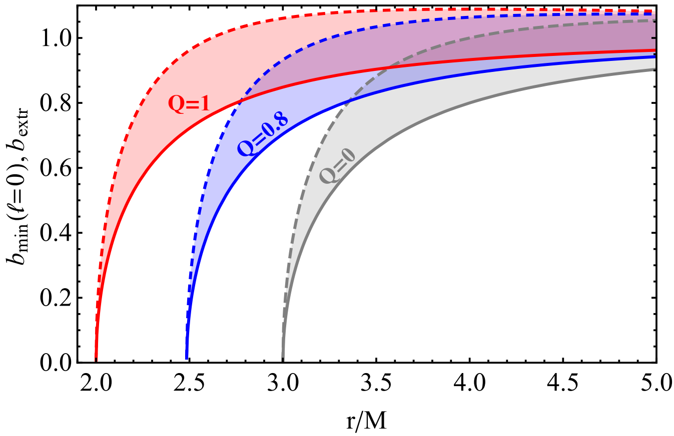

Figure 5. The radial profiles of minimal value of magnetic interaction parameter of free falling magnetized particles and extreme values of the interaction parameter are shown for the different values of electric charge of RN black hole. Unit of the electric charge is given in mass M.

light-red colored areas correspond to the values of electric charge of the black hole $Q = 0$, $Q/M = 0.8$ and $Q/M = 1$, respectively. One can see from the Fig. 5 that the inner position of circular orbits shifts to the center of black hole.

Thus, the extreme value of the parameter b corresponds to maximum value of the critical stable circular orbits $r_{\rm max}$ and it can be found through the solution of the following equation with respect to r

$$b_{\rm ext}(r; Q) = b \, . \tag{40}$$

The minimum value for the circular stable orbits can be found solving the following equation with respect to r,

$$b_{\rm min}(r; Q)|_{l=0} = b \, . \tag{41}$$

The distance between maximum and minimum radius of circular stable orbits $\Delta r = r_{\rm max} - r_{\rm min}$ give us the allowed area of the stable orbits for a magnetized particle. That means the circular stable orbits of a magnetized particle with the given interaction parameter b are confined in the range $r_{\rm max}(b; Q) > r > r_{\rm min}(b; Q)$. However, one can see from the equations (37) and (39) it is quite complicated to obtain the analytic solutions of equations (40) and (41). We solve the equation numerically and present the results in a table form.

The area of stable circular orbits of magnetized particles for the different values of electric charge of RN black hole is presented in Table 1 corresponding to the angular moment from 0 to $l_{\rm min}$. One can see from the table that the area is more dependent from interaction parameter than from electric charge of RN black hole.

One may express the dependence of minimum values of the specific angular momentum on magnetic coupling parameter β solving by the equation $b = b_{\rm min}$ with respect to the

Q/M	$b = 0.1$	$b = 0.5$	$b = 0.8$	$b = 0.95$	$b = 1$
0.1	0.004195	0.1346	0.6517	8.18971	–
0.3	0.004119	0.1322	0.640777	8.0494	–
0.5	0.003961	0.1273	0.61802	7.7598	–
0.8	0.003536	0.1139	0.5575	6.9981	–
1	0.003155	0.1023	0.50436	6.2388	–

Table 1. $\Delta r = r_{\max} - r_{\min}$ as function of the magnetic interaction parameter b and electric charge of RN black hole. The values of the range Δr are given in the units of $1.5(M/M_{\odot})$ km.

specific angular momentum l

$$l^2_{\min}(r; b, Q) = \frac{r\left(Q^2 - Mr\right)}{[r(r-3M)+2Q^2]^{3/2}} \times \left\{b\left(r(r-2M)+Q^2\right) - r\sqrt{r(r-3M)+2Q^2}\right\}. \tag{42}$$

Now one can easily get the dependence of the minimum value of specific energy inserting Eq. (42) into Eq. (36):

$$\mathcal{E}^2_{\min}(r; b, Q) = \frac{\sqrt{Mr - Q^2}\left[r(r-2M)+Q^2\right]}{r\,[r(r-3M)+2Q^2]^{3/2}} \times \left\{r\sqrt{r(r-3M)+2Q^2} - b\left[r(r-2M)+Q^2\right]\right\}. \tag{43}$$

3 CONCLUSION

In this paper, we have explored the dynamics of magnetized particles in the vicinity of an electrically charged RN black hole immersed in an external asymptotically uniform magnetic field using the Hamilton-Jacobi equation.

In the case of an electrically charged RN black hole due to the absence of interaction between the electric charge of the black hole and dipole moment of magnetized particles, we have assumed that the black hole is immersed in an external asymptotically uniform magnetic field. We have shown that the value of the magnetic coupling parameter for circular orbits and the energy which the magnetic coupling parameter minimum increase as the increase of the charge of RN black hole, while the minimum value of the parameter b decreases.

The range, where stable orbits are allowed, for the magnetized particle with the value $b_{extr} < 1$, narrows for the bigger values of the black hole charge and it becomes wider for the particle with the larger magnetic coupling parameter. When the magnetic coupling parameter of the particle is bigger than 1, the stable orbits do not exist around a black hole in the external magnetic field. The result of the study may be helpful for the estimations of the upper limits of the external magnetic field value around the black hole in the considerations of pulsars (neutron stars) as (dipolar) magnetized particles orbiting around the central supermassive black hole.

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