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Exact solutions of Einstein field equations

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ABSTRACT
We study the general form of spacetime metric representing gravitating axially symmetric compact objects. The properties, such as energy momentum tensor and interior and exterior geometry, of such objects are discussed. Due to the complex nature of gravitational field equations, especially interior of axial symmetric objects, we consider exact solutions of the special case of spherical symmetry object.

Keywords: Einstein field equations – exact solutions

1 INTRODUCTION

Recently discovered dark matter and dark energy in the universe has lead to construction of various modified theories of gravity as being alternate to the Einstein general theory of Relativity. These extended theories of gravity are likely to provide new exact solutions for (GR) the gravitational objects and from this point of view it is becoming extremely important to parametrize solutions of gravitational field equations. Most popular among them is Johannsen and Psaltis parametrization (Johannsen and Psaltis, 2011) and in recent years there were several attempts in this direction (see for example (Rezzolla and Zhidenko, 2014; Konoplya et al., 2016)). Following the parameterization of Rezzolla and Zhidenko (Rezzolla and Zhidenko, 2014), recently, the general parametrization for spherically symmetric and asymptotically flat black-hole spacetimes has been developed in an arbitrary metric theory of gravity (Konoplya et al., 2020). The exact axisymmetric and static solution of the Einstein equations coupled to the axisymmetric and static gravitating scalar field

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has also been investigated recently (Turimov et al., 2018). Later on, an anisotropic version of the well-known Tolman VII solution has been presented as stated by the gravitational decoupling by the minimal geometric deformation approach and this leads to determine an exact and physically acceptable interior two-fluid solution representing behavior of compact objects (Hensh and Stuchlík, 2019). There is also investigation that presents an exact solution describing the Schwarzschild-like black hole surrounded by the dust cosmological background for spherically symmetric dust distribution (Jaluvkova et al., 2017).

In this context, our interest in this paper is to write and discuss general expression for the axial spactime metric which could be valid for different theories of gravity. We study the general of the spactime metric of axially symmetric gravitational compact objects and test whether it is possible to have interior solution leading to the formation of naked singularity.

The first major work on study of singularities in GR dates back to 1965 when Penrose presented his seminal work on singularity theorems (Penrose, 1965). This theorem, which was later called the Penrose-Hawking singularity theorem, implies that the occurrence of singularities in GR is inevitable as long as matter obeys certain energy conditions. The super-dense regions of matter arising from gravitational collapse could be hidden from the outside observer giving rise to a black hole or it could be visible leading to a naked singularity (Hawking and Ellis, 1973). Penrose-Hawking theorem investigated the conditions that give rise to the emergence of singularities in GR (Hawking and Penrose, 1970). The presence of these singularities represent a breakdown of Einstein's theory as they give rise to the notion of geodesic incompleteness. Although the Penrose-Hawking theorem proved the existence of singularities, it did not shed light on the nature of the singularities arising in General Relativity as the theory allows both types of singularity to form in a scenario of gravitational collapse depending on the initial data from which the collapse develops. In fact, the occurrence of naked singularities in nature has been so far considered the limit of Einstein's theory and poses serious theoretical challenges as it indicates the breakdown of predictability in physics. This in turn led to the formulation of the cosmic censorship hypothesis proposed by Penrose (Penrose, 1969) in 1969 for validity of the Einstein gravity, preventing the singularity from being seen for observers outside. Irrespective of the fact that the cosmic censorship conjecture has not been proven yet, there have been, however, a large amount of work done in this context (see, e.g. Jacobson and Sotiriou, 2010; Saa and Santarelli, 2011; Li and Bambi, 2013; Düztaş et al., 2020; Barausse et al., 2010; Rocha and Cardoso, 2011; Shaymatov et al., 2015; Sorce and Wald, 2017; Shaymatov et al., 2019, 2020b; Gwak, 2018; Shaymatov et al., 2020a; Jiang and Zhang, 2020; Yang et al., 2020).

2 AXIALLY SYMMETRIC GRAVITATIONAL COMPACT OBJECTS

The axial symmetry, e.g., rotation of the central gravitational object, makes the gravitational field equations very complicated to obtain their exact solutions. In fact, the well-known Kerr (Kerr, 1963), Carter (Carter, 1968) and the Kerr-de Sitter (Carter, 1973) spacetime metrics have been known as external vacuum solutions, and they refer to partial solutions and correspond to a special kind of gravitational source. These solutions are associated with the gravitational field of a rotating uncharged or charged black hole, respectively (Carter, 1971). However, the exact interior solutions of gravitational field equations that can serve as the basis for any significant physical model of a rotating body have not yet been obtained. Hence, possible non-trivial results related to the interior solution being established even before solving the Einstein equations on the basis of only the equations of motion represent a definite value.

The established nature of pure rotation of the source having only single axis means that the quadratic form has axial symmetry and does not depend on time. Therefore, there is a frame of reference in which coordinates x^0, r, θ, and φ can be entered, thereby reflecting the time and axial symmetry of the metric in an explicit form. Having assumed precisely this nature of the coordinates and the frame of reference, in the general case, both in the inner and outer regions, the quadratic form for line element can be given as follows (Arifov, 1983):

$$\mathrm{d}s^2 = -(\mathrm{d}x^0)^2 + D(\mathrm{d}x^0 + E\sin^2\theta\mathrm{d}\phi)^2 + F\mathrm{d}r^2 + G\mathrm{d}\theta^2 + H\sin^2\theta\mathrm{d}\phi^2, \quad (1)$$

where D, E, F, G, and H are unknown functions depending only from r and θ coordinates.

The rotating black hole metric in Boyer-Lindquist coordinates, in particular, can be reduced to the following form

$$\mathrm{d}s^2 = -(\mathrm{d}x^0)^2 + \frac{\chi r}{r^2 + a^2\cos^2\theta}\left(\mathrm{d}x^0 + a\sin^2\theta\mathrm{d}\phi\right)^2 + \frac{r^2 + a^2\cos^2\theta}{r^2 - \chi r + a^2}\mathrm{d}r^2 + \left(r^2 + a^2\cos^2\theta\right)\mathrm{d}\theta^2 + \left(r^2 + a^2\right)\sin^2\theta\mathrm{d}\phi^2\,, \quad (2)$$

with χ being a constant having same meaning as in the Schwarzschild solution, with constant a is related to the total angular momentum of the rotating massive body.

At this stage we write the non-zero components of the metric tensor corresponding to the form (1) as

$$g_{00} = -(1-D)\,,\ g_{03} = DE\sin^2\theta\,,\ g_{11} = F\,,\ g_{22} = G\,,$$
$$g_{33} = \left(DE^2\sin^2\theta + H\right)\sin^2\theta\,, \quad (3)$$

and non-zero inverse components are

$$g^{00} = -\frac{1}{N}\left(DE^2\sin^2\theta + H\right),\ g^{03} = \frac{1}{N}DE\,,\ g^{11} = \frac{1}{F}\,,\ g^{22} = \frac{1}{G}\,,$$
$$g^{33} = \frac{1}{N\sin^2\theta}(1-D)\sin^{\theta}\,,\ g = -NGF\sin^2\theta\,, \quad (4)$$

with $N = DE^2\sin^2\theta + H(1-D)$.

It is worth noting that the axial symmetry of the spacetime metric (1) allows its simplification. Using two arbitrary coordinate transformation functions

$$r \to r' = r'(r,\theta)\,, \quad \text{and} \quad \theta \to \theta' = \theta'(r,\theta)\,, \quad (5)$$

one of the functions D, E, F, G and H, or any combination of them can be reduced to a predetermined function, while maintaining the orthogonality of the transformed r' and θ'

axes, namely the equality of the component g'^{12}. Further simplification is no longer possible. In the general case, the symmetry of the problem requires finding four independent functions included in the metric (1) and depending on two arguments.

If we introduce the notation for the derivatives

$$X' = \frac{\partial X}{dr} \quad \text{and} \quad \dot{X} = \frac{\partial X}{d\theta}\,, \tag{6}$$

the non-zero components of the Christoffel symbols take the form:

$$\begin{aligned}
\Gamma^0_{01} &= \frac{1}{2N}\left(D^2EE'\sin^2\theta - HD'\right), \Gamma^1_{00} = -\frac{D'}{2F}\,,\\
\Gamma^0_{02} &= \frac{1}{2N}\left[\left(\dot{E}\sin\theta + 2E\cos\theta\right)D^2E\sin\theta - H\dot{D}\right], \Gamma^2_{00} = -\frac{\dot{D}}{2G}\,,\\
\Gamma^0_{13} &= \frac{\sin^2\theta}{2N}\left[D^2E^2D'\sin^\theta + DEH' - H\,(DE)'\right],\\
\Gamma^0_{23} &= \frac{\sin^2\theta}{2N}\left[\left(\dot{E}\sin^\theta + 2E\cos\theta\right)D^2E^2\sin\theta + DE\dot{H} - H\left(\dot{DE}\right)\right],\\
\Gamma^1_{03} &= -\frac{\sin^2\theta}{2F}\,(DE)'\,, \Gamma^2_{03} = -\frac{\sin\theta}{2G}\left[\left(\dot{DE}\right)\sin\theta + 2DE\cos\theta\right],\\
\Gamma^3_{01} &= -\frac{1}{2N}\left[(DE)' - D^2E'\right], \Gamma^1_{11} = \frac{F'}{2F}\,, \Gamma^1_{12} = \frac{\dot{F}}{2F}\,,\\
\Gamma^3_{02} &= \frac{1}{2N}\left[\left(\dot{DE}\right) - D^2\dot{E} + 2\,(1-D)\,DE\cot\theta\right], \Gamma^1_{22} = -\frac{G'}{2F}\,,\\
\Gamma^1_{33} &= -\frac{\sin^2\theta}{2F}\left[\left(DE^2\right)'\sin^2\theta + H'\right], \Gamma^2_{11} = -\frac{\dot{F}}{2G}\,, \Gamma^2_{22} = \frac{\dot{G}}{2G}\,, \Gamma^2_{12} = \frac{G'}{2G}\,.
\end{aligned} \tag{7}$$

A frame of reference, in which the quadratic form for a rotating body can be reduced to the one in (1), is characterized by a complex motion of its components. Their absolute acceleration, $w_\mu = u_{\mu;\nu}u^\nu$ (where u^μ is the 4-velocity), is given by

$$w_0 = w_3 = 0\,, \; w_1 = -\frac{D'}{2\,(1-D)} \quad \text{and} \; w_2 = -\frac{\dot{D}}{2\,(1-D)}\,, \tag{8}$$

which are everywhere orthogonal to the family of hypersurfaces $D = const$. The non-zero components of the rotation tensor, $\mathbf{A} = \frac{1}{2}\left(u_{\mu,\nu} - u_{\nu,\mu} + u_\mu w_\nu - u_\nu w_{mu}\right)$, take the forms:

$$\begin{aligned}
\mathbf{A}_{oi} &= \mathbf{A}_{12} = 0\,, \; \mathbf{A}_{13} = \frac{1}{2}\frac{(DE)' - D^2E'}{(1-D)^{3/2}}\sin^2\theta\,,\\
\mathbf{A}_{23} &= \frac{1}{2}\frac{\left(\dot{DE}\right) - D^2\dot{E}}{(1-D)^{3/2}}\sin^2\theta + \frac{DE}{\sqrt{1-D}}\sin\theta\cos\theta\,.
\end{aligned} \tag{9}$$

The family of hypersurfaces being everywhere orthogonal to the direction of rotation components of the reference frame satisfies the following equation:

$$\frac{dr}{d\theta} = \frac{\mathbf{A}_{13}G}{\mathbf{A}_{23}F}\,. \tag{10}$$

For Kerr metric, in particular, the solutions of this equation belongs to the family

$$r^2 + a\cos\theta\,(a\cos\theta - const) = 0\,,\ \theta \neq \frac{\pi}{2}\,, \tag{11}$$

with equatorial hypersurface $\theta = \pi/2$.

Let the internal state of the source be described by the energy-momentum tensor $T^{\mu\nu} = (\rho + p)\,u^\mu u^\nu + pg^{\mu\nu}$ of an ideal fluid, the pressure and energy density. In the coordinates given in (1), we have the 4-velocity components u^1 and u^2 of the source being equal to zero, and if we introduce the following notation

$$\frac{\mathrm{d}\phi}{\mathrm{d}x^0} = \omega(r,\theta)\,, \tag{12}$$

then non-zero component becomes

$$u^3 = \frac{\mathrm{d}\phi}{\mathrm{d}\sigma} = \omega u^0\,, \tag{13}$$

where σ refers to the source's proper time.
From the above 4-velocity components $u^\mu\{u^0, 0, 0, \omega u^0\}$ and $u_\mu\{u_0, 0, 0, u_3\}$ respectively read

$$u^0 = \sqrt{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}\,, \tag{14}$$

and

$$u_0 = -\frac{1 - D\left(1 + \omega E\sin^2\theta\right)}{\sqrt{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}}\,,$$

$$u_3 = -\frac{DE\sin^2\theta\left(1 + \omega E\sin^2\theta\right) + \omega H\sin^2\theta}{\sqrt{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}}\,. \tag{15}$$

The internal state of the source is determined by three functions ρ, p and ω as a function of r and θ, and two of them are independent, i.e. ρ (or p) and ω. From all type of axial rotation bodies, solid-body rotation must be distinguished in the case in which $a = const$. In this case and in its own reference frame, both conditions imposed on the quadratic form, namely, stationarity and axial symmetry, can be expressed explicitly. Indeed, the transition $\phi \to \phi + \omega x^0$ to its own reference frame, in which $u^3 = 0$, does not change the quadratic forms (1). This is due to the fact that the relative distances between elements of a source rotating as a solid body remain unchanged. If the rotation of the gravitating object maintaining axial symmetry does not obey the solid angle law, then in its own frame of reference the metric can preserve axial symmetry in an explicit form, but can lose the explicit expression of the stationarity property. The metric already depends on time in its own frame of reference. The change in the relative distances between the elements of the source at constant coordinates assumes the metric tensor in its own reference frame. Equations (14) and

(15) retain their form for solid body rotation and in own reference frame if one formally sets $\omega = 0$.

At this stage we find out what restrictions are imposed by the equations of motion, $T^{\mu\nu}{}_{;\nu} = 0$, on the internal functions ρ, p and ω of the rotating body. Two of the four equations of motion corresponding to the coordinates x^0 and φ are satisfied identically. The other two equations are given by:

$$p' = \frac{1}{2}(\rho + p)\frac{D' + 2\omega\,(DE)'\sin^2\theta + \omega^2\left(DE^2\sin^2\theta + H\right)'\sin^2\theta}{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}, \tag{16}$$

$$\dot{p} = \frac{1}{2}(\rho + p)\frac{\dot{D} + 2\omega\dot{\left(DE\sin^2\theta\right)} + \omega^2\left[\dot{\left(DE^2\sin^2\theta + H\right)}\sin^2\theta\right]}{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}. \tag{17}$$

The above equations can be rewritten as follows:

$$\begin{aligned}\frac{\mathrm{d}p}{\rho + p} =& \mathrm{d}Log\left[1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^\theta\right]^{-1/2} \\ &- \frac{DE\sin^2\theta\left(1 + \omega E\sin^2\theta\right) + \omega H\sin^2\theta}{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}d\omega\,.\end{aligned} \tag{18}$$

From above the left side is, according to the thermodynamic equation of state, the total differential function

$$\int\frac{\mathrm{d}p}{\rho + p}, \tag{19}$$

which is solved further for an incompressible ideal fluid and since the first term on the right is also a total differential, the second term on the right can then only be a total differential of some function r and θ in the case of $\omega(r,\theta) \neq const$. This would be possible if and only if the factor in front of $d\omega$ depends on ω and does not explicitly depend on r and θ, i.e.,

$$\frac{DE\sin^2\theta\left(1 + \omega E\sin^2\theta\right) + \omega H\sin^2\theta}{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta} = b(\omega)\,, \tag{20}$$

where $b(\omega)$ is an arbitrary function. The above equation (20) establishes an algebraic relationship between four functions of coordinates r and θ, i.e, ω, $(1 - D)$, $DE\sin^2\theta$ and $\left(DE\sin^2\theta + H\right)\sin^2\theta$. However, in the case of rotation of the gravitating body, there appears no such dependence. The equations of motion are thus given by,

$$\exp\left[\int\frac{\mathrm{d}p}{\rho + p} + \int b(\omega)\mathrm{d}\omega\right] = \frac{const}{\sqrt{1 - D\left(1 + \omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}}, \tag{21}$$

if $\omega(r,\theta) \neq const$ while

$$\exp\left[\int \frac{\mathrm{d}p}{\rho+p}\right] = \frac{const}{\sqrt{1 - D\left(1+\omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}}\,, \tag{22}$$

if $\omega = const$.

For an incompressible ideal fluid $\rho = const$, for example, the equations of motion are completely integrated by

$$\rho + p = \frac{const}{\sqrt{1 - D\left(1+\omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta}}\,. \tag{23}$$

The hypersurface, on which the pressure is constant, is called equipotential. The section of the equipotential hypersurface of the coordinate hyperpsurface $x^0 = const$ obviously refers to the closed surface. Equipotential hypersurfaces form, according to (21-22), a one-parameter family.

Theorem 1

The boundary of an axially rotating body is an equipotential hypersurface, on which the pressure is zero. The shape of the border is determined by

$$\left[1 - D\left(1+\omega E\sin^2\theta\right)^2 - \omega^2 H\sin^2\theta\right]\exp\left\{-2\int b(\omega)d\omega\right\} = const\,, \tag{24}$$

if $\omega(r,\theta) \neq const$ while

$$D\left(1+\omega E\sin^2\theta\right)^2 + \omega^2 H\sin^2\theta = const\,, \tag{25}$$

in the case of solid body rotation.

A certain correspondence can be established between the distribution functions of pressure and mass density of rotating and non-rotating bodies.

Theorem 2

For each given equation of state of matter of an axially rotating body and given distribution of the angular velocity ω in the quadratic frame of reference (1) there exists such a coordinate grid that the distribution functions of pressure, density of the number of particles, and density of mass-energy coincide with the corresponding functions of a non-rotating body in a quadratic reference frame, and the boundaries of the body are coordinate hypersurfaces $r = const$.

3 CONCLUSIONS

In this work, we have discussed general form of axial symmetric spacetime which could be applied to the possible solutions of field equations in various extended theories of gravity.

We have seen in the above that the general solution of Einstein's equations for a stationary axially symmetric source, the equation of state and the distribution of the angular velocity of rotation of the substance given, and the boundaries being free correspond to two types of the structure of the source. The first type of sources represents only one external solution having free boundary for which pressure and density of mass and number of particles take a maximum value in the center and fall monotonically towards the boundary. Another type of sources has two, internal and external, boundaries at which the pressure is equal to zero; a cavity free from matter and thermal radiation from the source, with a singular time-like world line in the center and the pressure and density of the mass and number of particles take maximum values at the critical hypersurface and fall monotonically towards both boundaries. As a consequence of the analysis we showed that it is possible to have only an external solution associated with rotation parameter and realized that it is however impossible to obtain interior solution in the case of rotation.

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