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Construction of Taub-NUT black hole solutions in general relativity coupled to nonlinear electrodynamics

Bobir Toshmatov,[1,2,3,a] Sardor Tojiev[1]
and Jakhongir Xomidjonov[4]

[1]Ulugh Beg Astronomical Institute, Astronomy str. 33, Tashkent 100052, Uzbekistan,
[2]Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Kori Niyoziy 39, Tashkent 100000, Uzbekistan
[3]Webster University in Tashkent, Alisher Navoiy 13, Tashkent 100011, Uzbekistan
[4]Physics Department, Ferghana Polytechnical Institute, Ferghana str. 86, Ferghana 150107, Uzbekistan
[a]toshmatov@astrin.uz

ABSTRACT
We present a formalism of construction of the Taub-NUT black hole (BH) solutions in general relativity (GR) coupled to the nonlinear electrodynamics (NED). We have shown that the constructed spacetimes can be electrically, magnetically or dyonically charged.

Keywords: Nonlinear electrodynamics – black holes – Taub-NUT spacetime

1 INTRODUCTION

It has been shown that the most general axially symmetric vacuum solution with separable equation of motion is the Kerr-NUT solution (Dadhich and Turakulov, 2002). Further it also turns out that the metric is invariant under the transformation $M \leftrightarrow il$, $r \leftrightarrow ia\lambda$, where λ is an angle coordinate. Under this duality transformation, it can interestingly be shown that Kerr solution is dual to massless Kerr-NUT solution (Nouri-Zonoz et al., 1999; Turakulov and Dadhich, 2001). The charged version of the general Kerr-NUT solution is the general Kerr-Newman-NUT solution, and this charge can only be electric and not magnetic. There is quite an extensive literature on NUT geometry in an attempt to understand its physical nature and properties. We would like to refer to an excellent review (Lynden-Bell and Nouri-Zonoz, 1998) critiquing all the earlier works as well as it makes a strong case for NUT parameter to be looked upon as gravomagnetic charge – a dual to gravoelectric charge mass. Of course it has a number of undesirable features such as it is not asymptotically flat and admits closed timelike curves. NUT parameter could by and large be considered as gravomagnetic charge (Mukherjee et al., 2019; Dadhich and Patel, 2002).

In the present paper we aim to develop formalism for construction of black hole solutions in the GR coupled to the NED in the Taub-NUT framework. The first of all, we seek for the possibility if the dyonically charged, i.e., electrically and magnetically charged at the same time, solutions can be constructed in this framework. If there is no such possibility, we solve the field equations for the electrically and magnetically charged spacetimes separately. This paper is organised as follows: in section 2 we present the main equations of motion of the system GR coupled to the NED. In section 3 the construction of the black hole solutions in GR coupled to the linear electrodynamics is presented in the Taub-NUT framework, while section 4 is devoted to the construction of the static, axially symmetric electrically and magnetically charged black hole solution in GR coupled to the NED is presented. Finally, in section 5 we summarize the results obtained in the paper. Throughout the paper, we adopt the following signature convention $(-,+,+,+)$ for the space-time metric and make use of natural units, thus setting $c=\hbar=G=1$.

2 BH SOLUTIONS COUPLED IN NED WITH NUT SYMMETRY

The action of Einstein's gravity coupled to the NED is given as

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,(R-L)\ , \tag{1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, R is the scalar curvature, and L represents the Lagrangian density of the NED field that is function of the EM field strength, $L=L(F)$, with $F=F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu}$ is the EM field tensor that can be written in terms of a gauge potential A_μ as $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. Definition of the EM field tensor shows that $F_{\mu\nu}$ is anti-symmetric and it has only six independent components.

By neglecting the EM sources, one can write the covariant equations of motion in the form

$$G_{\mu\nu} = T_{\mu\nu}\ , \tag{2}$$

$$\nabla_\nu\left(L_F F^{\mu\nu}\right) = 0\ , \tag{3}$$

where the Einstein tensor $G_{\mu\nu}=R_{\mu\nu}-Rg_{\mu\nu}/2$ and $T_{\mu\nu}$ is the energy-momentum tensor of the EM field determined by the relation

$$T_{\mu\nu} = 2\left(L_F F_\mu^{\alpha}F_{\nu\alpha}-\frac{1}{4}g_{\mu\nu}L\right)\ , \tag{4}$$

where $L_F=\partial_F L$.

The line element of the static, axially symmetric Taub-NUT BH reads

$$\mathrm{d}s^2 = -\frac{\Delta}{\Sigma}\left(\mathrm{d}t-\chi\mathrm{d}\phi\right)^2+\Sigma\left(\frac{\mathrm{d}r^2}{\Delta}+\mathrm{d}\theta^2\right)+\Sigma\sin^2\theta\mathrm{d}\phi^2\ , \tag{5}$$

where

$$\Delta = r^2 - 2m(r)r - l^2, \quad \Sigma = r^2 + l^2, \quad \chi = -2l\cos\theta,$$

The Carter tetrad of 1-forms for the spacetime matric (5) is written as (Znajek, 1977)

$$\begin{aligned} \omega^t &= \sqrt{\frac{\Delta}{\Sigma}}(\mathrm{d}t - \chi \mathrm{d}\phi)\,, \quad \omega^r = \sqrt{\frac{\Sigma}{\Delta}}\mathrm{d}r\,, \\ \omega^\theta &= \sqrt{\Sigma}\mathrm{d}\theta\,, \quad \omega^\phi = \sqrt{\Sigma}\sin\theta \mathrm{d}\phi\,. \end{aligned} \tag{6}$$

The non-zero components of the Einstein tensor are given by

$$\begin{aligned} G_{tt} &= \frac{2r^2\Delta m'}{\Sigma^3}, \qquad G_{t\phi} = -\frac{2\chi r^2 \Delta m'}{\Sigma^3}, \\ G_{rr} &= -\frac{2r^2 m'}{\Sigma\Delta}, \qquad G_{\theta\theta} = -\frac{2l^2 m'}{\Sigma} - rm'', \\ G_{\phi\phi} &= \frac{2r^2\Delta m'}{\Sigma^3}\chi^2 - \left(\frac{2l^2 m'}{\Sigma} + rm''\right)\sin^2\theta. \end{aligned} \tag{7}$$

Here we have the following relation among components of the Einstein tensor:

$$G_{\phi\phi} = G_{\theta\theta}\sin^2\theta + G_{tt}\chi^2\,. \tag{8}$$

One can easily notice that if the mass is constant, $m(r) = M$, then, the all the components vanish and we end up with the Schwarzschild Taub-NUT solution. Now we are going to obtain some solutions by coupling general relativity in Taub-NUT framework with NED which can be electrically or magnetically charged.

3 IN LINEAR ELECTRODYNAMICS

The linear electrodynamics is defined by the Maxwell theory which is characterized by the lagrangian density

$$L = F\,, \tag{9}$$

namely, $L_F = 1$. If the spacetime has electric charge Q_e and magnetic charge Q_m, in the linear electrodynamics the 4- potential of the EM field can be expressed by the 1-form of Carter tetrad (6) (Znajek, 1977) as

$$A = -\frac{Q_e r}{\sqrt{\Sigma\Delta}}\omega^t - \frac{Q_m\cot\theta}{\sqrt{\Sigma}}\omega^\phi, \tag{10}$$

i.e.,

$$A = -\frac{Q_e r}{\Sigma}(\mathrm{d}t - \chi \mathrm{d}\phi) - Q_m\cos\theta \mathrm{d}\phi. \tag{11}$$

3.1 Electrically charged spacetime

If we consider the spacetime is electrically charged, then the vector potential (11) takes the form:

$$A_\mu = -\frac{Q_e r}{\Sigma}\delta^t_\mu - \frac{2Q_e r l\cos\theta}{\Sigma}\delta^\phi_\mu, \tag{12}$$

Then, we have the following non-zero covariant components of the EM field tensor:

$$F_{tr} = \frac{Q_e\left(l^2-r^2\right)}{\Sigma^2}, \quad F_{r\phi} = \frac{2lQ_e\cos\theta\left(r^2-l^2\right)}{\Sigma^2},$$
$$F_{\theta\phi} = \frac{2lQ_e r\sin\theta}{\Sigma}\ . \tag{13}$$

The non-zero contravariant components of the EM field tensor are found by the relation $F^{\mu\nu} = g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}$ and are given by:

$$F^{tr} = \frac{Q_e\left(r^2-l^2\right)}{\Sigma^2}, \quad F^{t\theta} = \frac{4l^2Q_e r\cot\theta}{\Sigma^3},$$
$$F^{\theta\phi} = \frac{2lQ_e r\csc\theta}{\Sigma^3}\ . \tag{14}$$

By combining covariant (13) and contravariant (14) Maxwell tensor, we obtain F as

$$F = -\frac{2Q_e^2\left(r^4-6l^2r^2+l^4\right)}{\Sigma^4}\ . \tag{15}$$

The nonvanoshing components of energy-momentum tensor are found from (4) as

$$T_{tt} = \frac{Q_e^2\Delta}{\Sigma^3}, \quad T_{t\phi} = -\frac{Q_e^2\chi\Delta}{\Sigma^3},$$
$$T_{rr} = -\frac{Q_e^2}{\Sigma\Delta}, \quad T_{\theta\theta} = \frac{Q_e^2}{\Sigma},$$
$$T_{\phi\phi} = \frac{Q_e^2}{\Sigma}\sin^2\theta + \frac{Q_e^2\Delta}{\Sigma^3}\chi^2\ , \tag{16}$$

Here we have an interesting relation

$$T_{\phi\phi} = T_{\theta\theta}\sin^2\theta + T_{tt}\chi^2\ . \tag{17}$$

that is symmetric counterpart of the relations in components of the Einstein tensor (8). Because of these symmetry, number of independent equations of the Einstein equations is decreased by one. Now by solving the Einstein equations (2) by using the non-zero componnets of Einstein tensor (7) and energy-momentum tensor of the electrically charged spacetime in linear electrodynamics (16), we obtain two differential equations $m' - Q_e^2/2r = 0$ and $m'' + Q_e^2/r^3 = 0$ which give the following general mass function:

$$m = M - \frac{Q_e^2}{2r}\ , \tag{18}$$

If we insert mass function (18) to the spacetime metric (5), it reduces to the electrically charged Reissner-Nordström-Taub-NUT solution.

3.2 Magnetically charged spacetime

If we consider the Taub-NUT spacetime (5) is magnetically charged in linear electrodynamics then, the 4-potential of the EM field is given as $A_\phi = -Q_m \cos\theta$. Then, only non-zero independent component of the EM tensor is $F_{\theta\phi} = Q_m \sin\theta$. And contravariant non-zero components of it are given as

$$F^{t\theta} = -\frac{Q\chi \csc\theta}{\Sigma^2}\ , \quad F^{\theta\phi} = \frac{Q \csc\theta}{\Sigma^2}\ . \tag{19}$$

The EM field strength or lagrangian density of Maxwell electrodynamics is

$$F = \frac{2Q_m^2}{\Sigma^2}\ , \tag{20}$$

The nonvanishing components of energy-momentum tensor are found from (4) and they are the same with the ones of the electrically charged case (16). Therefore, we will not repeat the calculations, instead we will give the final result which the mass function of the magnetically charged Taub-NUT solution in linear electrodynamics is given as

$$m = M - \frac{Q_m^2}{2r}\ , \tag{21}$$

and it represents the magnetically charged Reissner-Nordström-Taub-NUT solution in linear electrodynamics.

4 IN NONLINEAR ELECTRODYNAMICS

In this section we consider more general case which is construction of electrically and magnetically charged solution of general relativity coupled to the NED in Taub-NUT framework. The nonlinear electrodynamics is defined by the lagrangian density which is nonlinear function of EM field strength F, i.e.,

$$L_F \equiv \frac{\partial L}{\partial F} \neq \text{Constant.} \tag{22}$$

Construction of the electrically and magnetically charged, spherically symmetric, asymptotically flat solutions of general relativity coupled to the NED have been presented by several authors (Ayón-Beato and García, 1998; Bronnikov, 2000; Burinskii and Hildebrandt, 2002; Fan and Wang, 2016; Bronnikov, 2017; Toshmatov et al., 2018a,b,c). Here, for the first time we present the construction electrically and magnetically charged, axially symmetric, asymptotically non-flat (Taub-NUT) solution in general relativity coupled to the NED.

4.1 Electrically charged solution

In this subsection we present the construction of electrically charged, axially symmetric non-flat Taub-NUT solution in the NED. Let us generalize the vector potential (11) as

$$A = \psi(r)\frac{r^2}{\Sigma}(\mathrm{d}t - \chi \mathrm{d}\phi)\ . \tag{23}$$

Covariant components of the EM tensor

$$F_{tr} = -\frac{r^2\psi'}{\Sigma} - \frac{2l^2 r\psi}{\Sigma^2}\,, \quad F_{r\theta} = -\frac{r^2\psi'\chi}{\Sigma} - \frac{2l^2 r\psi\chi}{\Sigma^2},$$
$$F_{\theta\phi} = -\frac{2lr^2\psi\sin\theta}{\Sigma}, \tag{24}$$

Contravariant components of the EM tensor

$$F^{tr} = \frac{r^2\psi'}{\Sigma} + \frac{2l^2 r\psi}{\Sigma^2}, \quad F^{t\theta} = \frac{2lr^2\psi\chi\csc\theta}{\Sigma^3},$$
$$F^{\theta\phi} = -\frac{2lr^2\psi\csc\theta}{\Sigma^3}\,, \tag{25}$$

The EM field strength of the electrically charged spacetime of the NED in the Taub-NUT geometry is given by

$$F = -\frac{2r^2}{\Sigma^2}\left[r^2\psi'^2 + \frac{4l^2 r\psi\psi'}{\Sigma} + \frac{4l^2(l^2-r^2)\psi^2}{\Sigma^2}\right]. \tag{26}$$

From (4) one finds the non-zero components the energy-momentum tensor of the NED as

$$T_{tt} = \Delta\left[\frac{2r^2L_F\left(2l^2\psi + r\Sigma\psi'\right)^2}{\Sigma^5} + \frac{L}{2\Sigma}\right],$$
$$T_{t\phi} = -\Delta\chi\left[\frac{2r^2L_F\left(2l^2\psi + r\Sigma\psi'\right)^2}{\Sigma^5} + \frac{L}{2\Sigma}\right],$$
$$T_{rr} = -\frac{\Sigma^2}{\Delta}\left[\frac{2r^2L_F\left(2l^2\psi + r\Sigma\psi'\right)^2}{\Sigma^5} + \frac{L}{2\Sigma}\right]\,, \tag{27}$$
$$T_{\theta\theta} = \frac{8l^2r^4L_F\psi^2}{\Sigma^3} - \frac{1}{2}\Sigma L,$$
$$T_{\phi\phi} = \left[\frac{8l^2r^4L_F\psi^2}{\Sigma^3} - \frac{1}{2}\Sigma L\right]\sin^2\theta + \Delta\left[\frac{2r^2L_F\left(2l^2\psi + r\Sigma\psi'\right)^2}{\Sigma^5} + \frac{L}{2\Sigma}\right]\chi^2,$$

Here again the relation (17) is satisfied. By substituting the Einstein (7) and energy-momentum (27) tensors into the Einstein equations (2), we obtain the following two independent equations:

$$\frac{2r^2L_F\left(2l^2\psi + r\Sigma\psi'\right)^2}{\Sigma^3} - \frac{2r^2m'}{\Sigma} + \frac{1}{2}\Sigma L = 0\,,$$
$$\frac{8l^2r^4L_F\psi^2}{\Sigma^3} + \frac{2l^2m'}{\Sigma} - \frac{1}{2}\Sigma L + rm'' = 0\,, \tag{28}$$

By solving equations (28) with respect to L and L_F, simultaneously, we arrive at the expressions

$$L = \frac{2rm''\left(2l^2\psi + r\Sigma\psi'\right)^2}{\Sigma\left[4l^2r^2\psi^2 + (2l^2\psi + r\Sigma\psi')^2\right]} + \frac{4l^2m'\left[\left(2l^2\psi + r\Sigma\psi'\right)^2 + 4r^4\psi^2\right]}{\Sigma^2\left[4l^2r^2\psi^2 + (2l^2\psi + r\Sigma\psi')^2\right]}, \tag{29}$$

$$L_F = \frac{\Sigma^2\left[2\left(r^2 - l^2\right)m' - r\Sigma m''\right]}{2r^2\left[4l^2r^2\psi^2 + (2l^2\psi + r\Sigma\psi')^2\right]}, \tag{30}$$

Moreover, from the conservation of charge that is defined by $\mu = t$ in equation (3), one obtains the following relation:

$$\left[rL_F\left(\frac{2l^2\psi}{\Sigma} + r\psi'\right)\right]' + \frac{4l^2r^2L_F\psi}{\Sigma^2} = 0\,, \tag{31}$$

If $l = 0$ is considered, then we arrive at the conservation of charge in the spherically symmetric spacetimes in GR coupled to the NED (Toshmatov et al., 2018c)

$$\left(r^2L_F\psi'\right)' = 0\,. \tag{32}$$

4.2 Magnetically charged solution

In this subsection we consider construction of magnetically charged Taub-NUT solution in the NED. Nonzero components of the EM field tensor of the magnetically charged NED are $F_{\theta\phi} = Q_m \sin\theta = -F_{\phi\theta}$. The EM field strength is

$$F = \frac{2Q_m^2}{(r^2 + l^2)^2}\,, \tag{33}$$

From (4) one finds the non-zero components the energy-momentum tensor of the NED as

$$\begin{aligned} T_{tt} &= \frac{L\Delta}{2\Sigma}, \qquad T_{t\phi} = -\frac{L\Delta\chi}{2\Sigma},\\ T_{rr} &= -\frac{L\Sigma}{2\Delta}, \qquad T_{\theta\theta} = \frac{2Q^2L_F}{\Sigma} - \frac{1}{2}L\Sigma,\\ T_{\phi\phi} &= \left(\frac{2Q^2L_F}{\Sigma} - \frac{1}{2}L\Sigma\right)\sin^2\theta + \frac{L\Delta\chi^2}{2\Sigma}. \end{aligned} \tag{34}$$

Here again we have the interesting relation (17). By substituting the Einstein (7) and energy-momentum (34) tensors into the Einstein equations (2), we obtain two independent equations

$$\frac{1}{2}\Sigma L - \frac{2r^2m'}{\Sigma} = 0,$$
$$\frac{2Q_m^2L_F}{\Sigma} - \frac{1}{2}\Sigma L + \frac{2l^2m'}{\Sigma} + rm'' = 0, \tag{35}$$

By solving equations (35), we obtain

$$L = \frac{4r^2m'}{\Sigma^2}, \tag{36}$$
$$L_F = \frac{2(r^2-l^2)m' - r\Sigma m''}{2Q_m^2}, \tag{37}$$

If we assume that the EM field is linear, i.e., the Maxwell field, $L = F$ and $L_F = 1$, and NUT charge parameter is equal to zero, $l = 0$, then, by solving the above equations we arrive at the mass function $m = M - Q_m^2/2r$ that represents again the Reissner-Nordström solution which is the solution of the Einstein-Maxwell equations. If $l \neq 0$, it represents the Reissner-Nordström-Taub-NUT spacetimes.

4.3 Dyonically charged solution

As in the previous subsections we have shown that the electrically and magnetically charged solutions can be obtained in GR coupled to the NED in the static spacetime with NUT symmetry, in the current subsection we consider if it is possible to construct the spacetime admitting both charges at the same time. To do so, we must solve the field equations (2) and (3) for the line element of the spacetime (5) with the 4-electromagnetic potential (11). Due to the cumbersome forms of equations, we do not report them here, but instead, we only present the results. Thus, solving the Einstein field equations, we obtain that to have the dyonically charged spacetime in GR coupled to the NED with NUT symmetry, the Lagrangian density of the NED must be related to the mass function of the spacetime via the following expressions:

$$L = \frac{2r^2\left(Am' + Bm''\right)}{\Sigma^3\left[\Sigma\left(Q_m^2 + r^4\psi'^2\right) + 4l^2r^2\psi^2 + 4lr^2\psi\left(lr\psi' - Q_m\right)\right]}, \tag{38}$$
$$L_F = \frac{2\left(r^4 - l^4\right)m' - r\Sigma^2m''}{2\left[\Sigma\left(Q_m^2 + r^4\psi'^2\right) + 4l^2r^2\psi^2 + 4lr^2\psi\left(lr\psi' - Q_m\right)\right]} \tag{39}$$

where

$$A = 2\left[\Sigma^2\left(l^2r^2\psi'^2 + Q_m^2\right) + 4l^2\left(l^4 + r^4\right)\psi^2 + 4lr\Sigma\psi\left(l^3\psi' - Q_mr\right)\right],$$
$$B = r\Sigma\left(r\Sigma\psi' + 2l^2\psi\right)^2,$$

Moreover, from the conservation of charge that is defined by $\mu = t$ in equation (3), one obtains the following relation:

$$\left[rL_F\left(\frac{2l^2\psi}{\Sigma}+r\psi'\right)\right]' - \frac{2l\left(Q_m\Sigma-2lr^2\psi\right)}{\Sigma^3} = 0\,, \tag{40}$$

If $l = 0$ is considered, then we again arrive at the conservation of charge in the spherically symmetric spacetimes in GR coupled to the NED (32) and it confirms the pioneering results in (Demianski et al., 1986; Mazharimousavi and Halilsoy, 2012). In the linear electrodynamics (L_F = const), that would give us the well-known Coulomb's potential $\psi = Q_e/r$.

5 CONCLUSION

In the present paper we demonstrated the formalism for construction of axially symmetric, static, asymptotically non-flat black hole solutions in GR coupled to the NED in the Taub-NUT framework. The presented formalism is easy to handle as by switching off the NUT parameter of the spacetime, one can smoothly turn to the spherically symmetric, asymptotically flat counterparts of the spacetimes, or by turning off the charge parameter, one can recover the well-known solutions of the GR coupled to the NED or linear electrodynamics. The formalism has shown that in the Taub-NUT framework the electrically, magnetically and dyonically charged spacetime can be constructed.

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