Neutron stars with quark cores

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ABSTRACT

We investigate physical conditions under which nuclear matter may cross to quark matter using several equations of state for both phases. We calculate the combined equation of state using the nuclear matter equation of state for low-density region and the quark matter equation of state for the high-density region using Maxwell construction. Then we use it to calculate properties of non-rotating compact stars with quark cores and hadronic surface. We focus primarily on the maximum mass of a non-rotating star and on the moment of inertia of quark core for different combinations of selected equations of state. This work is the starting point for future investigation of rotating neutron stars with quark cores.

Keywords: Neutron stars – phase transitions – hybrid stars

1 INTRODUCTION

Neutron stars are the densest objects with internal structure currently known to exist in the Universe. Their structure is governed by general relativity and nuclear physics of very dense matter (densities in the cores of neutron stars can reach values several times higher than standard nuclear matter density). The matter in the cores of neutron stars is in the standard picture composed of neutrons, protons, and electrons in β-equilibrium, however, at sufficiently high densities the matter can undergo the phase transition to deconfined quarks.

Phase transitions in neutron stars are of huge interest since they can affect the global properties of neutron stars like the neutron star mass M, radius R moment of inertia I or the Love numbers Λ. For the current status of phase transition in compact stars see the recent overview by Blaschke and Chamel (2018).

Since phase transition to quark matter corresponds to the transition to a form of matter that is energetically preferable at high densities, it leads to the softening of the equation of state in that region. Equation of state with phase transition describing hybrid stars should therefore allow for the maximum mass that is smaller than the maximum mass allowed by the hadronic equation of state without phase transition. Therefore observations of massive neutron stars constrain the hadronic equation of state even if it does not describe the whole interior of the observed star. Currently, the most massive neutron stars known are $M = 2.01 \pm 0.04 M_{\odot}$ by Antoniadis et al. (2013) and $M = 1.97 \pm 0.04$ by Demorest et al.

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(2010) that are both in a binary system with a white dwarf, and massive enough to put serious constraints on the equation of state of neutron star matter. Other observational constraints on equations of state with phase transition have been discussed also by Kurkela et al. (2014) who shown that constraints from neutron star observations on the equation of state of neutron star matter are insensitive to the size of quark matter core. The maximum mass of neutron stars with quark cores was also discussed by other authors - see e.g. Zdunik and Haensel (2013).

In this short presentation, we focus on simple calculations using several equations of state of hadronic matter and for each, we calculate the physical conditions of Maxwell phase transition to simple MIT Bag model with various values of bag constant. We calculate non-rotating models of compact stars with quark core and focus on mass-radius relation, on maximum mass, and on the moment of inertia of quark core.

2 MODEL

2.1 Equations of state

Hadronic EoS: In our presentation we use selection of representative hadronic equations of state namely APR (Akmal et al., 1998), FPS (Lorenz et al., 1993), Gandolfi (Gandolfi et al., 2010), KDE (Agrawal et al., 2005), NRAPR (Steiner et al., 2005), SLy4 Rikovska Stone et al. (2003), and UBS (Urbanec et al., 2010). Each of these equations of state is composed of an equation of state describing nuclear matter composed of neutrons, protons, electrons, and muons in β-equilibrium based on various theoretical models of nucleon-nucleon interaction and are matched to a standard set of equations of state describing the low-density region, where the matter is composed of stable atomic nuclei or free neutrons in equilibrium with nuclei (Baym et al., 1971).

Quark EoS: We assume the quark core to consist of mass-less u and d quarks. To describe quark matter we use MIT Bag model (Chodos et al., 1974; Farhi and Jaffe, 1984; Haensel et al., 1986) where pressure P is related to energy density ρ by

$$P = \frac{1}{3}(\rho - 4B), \tag{1}$$

where B is Bag constant that gives energy density corresponding to zero pressure $\rho_0 = 4B$. The factor $1/3$ can be related to sound speed of quark matter $v_\mathrm{s} = c\sqrt{\mathrm{d}P/\mathrm{d}\rho} = c/\sqrt{3}$. The baryon number density is given as

$$n_\mathrm{B} = \left[\frac{4(1-2\alpha_\mathrm{c}/\pi)^{1/3}}{9\pi^{2/3}\hbar}(\rho - B)\right]^{3/4}, \tag{2}$$

where α_c is strong interaction coupling constant. Chemical potential per baryon is given by

$$\mu_\mathrm{B} = \frac{\rho + P}{n_\mathrm{B}}. \tag{3}$$

In our calculations we take $\alpha_\mathrm{c} = 0$ and for bag constant we choose six different values $B = \{2; 2.5; 3; 3.5; 4; 4.5\} \times 10^{14}\mathrm{g.cm}^{-3}$.

Phase transition For all possible combinations of hadronic EoS and quark EoS, we calculate pressure and baryonic chemical potential of the phase transition using Maxwell construction, i.e. the resulting EoS has continuous chemical potential as a function of pressure $\mu_B = \mu_B(P)$. Phase transition takes place at pressure P_{pt} and baryonic chemical potential μ_{pt} that is calculated for each combination of the hadronic equation of state and quark equation of state from our selection. The same approach to model the phase transition was used e.g. by Benić et al. (2015) or by Alvarez-Castillo et al. (2019) where they used more advanced quark EoS based on QCD and used relativistic mean-field model EoS of hadronic matter.

Compact star models Global properties of non-rotating compact stars are given by differential equations of hydrostatic equilibrium - TOV equation (Tolman, 1939; Oppenheimer and Volkoff, 1939)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{(\rho + P)\left[m(r) + 4\pi r^3 P\right]}{r\left[r - 2m(r)\right]}, \tag{4}$$

where $m(r)$ is mass inside a sphere of radius r and is given by

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi\rho r^2. \tag{5}$$

The set of differential equation is solved for given value of central pressure. Equations are integrated while the pressure remains positive and the radius r where pressure vanishes is giving the surface of compact star, i.e. $P(R) = 0$ with R being the radius of the compact star. Mass is given by $M = m(R)$. We also calculate moment of inertia of the star I_{tot} given by (Hartle, 1967)

$$I_{tot} = -\frac{2}{3}\int_0^R r^3 \left(\frac{\mathrm{d}j}{\mathrm{d}r}\right)\left(\frac{\tilde{\omega}}{\Omega}\right)\mathrm{d}r, \tag{6}$$

where Ω is angular velocity of the star and j is given by $j = 1/\sqrt{-g_{rr}g_{tt}}$. The function $\tilde{\omega}$ is found by solving equation

$$\frac{1}{r^4}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 j \frac{\mathrm{d}\tilde{\omega}}{\mathrm{d}r}\right) + \frac{4}{r}\frac{\mathrm{d}j}{\mathrm{d}r}\tilde{\omega} = 0. \tag{7}$$

One can find a moment of inertia of the quark core I_{core} by performing the integral in eq. (6) to $r_{pt} = r(P = P_{pt})$ instead performing the integral to the surface where $r = R$.

3 RESULTS AND DISCUSSION.

We solved the structure equations described in the previous section to obtain mass, radius, a moment of inertia of the whole star, and the moment of inertia of the quark core as a function of central pressure. At first, we solved the problem for purely hadronic equations of state and we present the mass-radius relation on the left panel of Fig. 1. The results when we assumed only quark matter described by the MIT Bag model is presented on the right

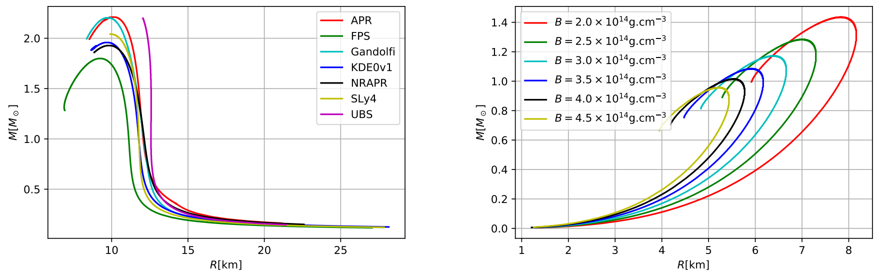

Figure 1. Mass vs radius of neutron stars with hadronic equations of state (left) and quark stars with MIT Bag model (right).

panel of Fig. 1. The maximum mass of quark stars is given purely by bag constant in a simple model we assumed here and was discussed by Haensel et al. (1986). They found a maximum mass and corresponding radius, a moment of inertia, and central density as a function of Bag constant (see eq. (28) in Haensel et al. (1986) and related discussion for details)[1].

Mass-radius relations of hybrid stars (neutron stars with quark cores) are discussed on Fig. 2 where each panel correspond to a particular value of bag constant and bag constant is increasing from the top left to bottom right. The lines at each panel starting on the right where mass is smallest and radius largest are corresponding to lower central pressures. As central pressure increases the radius is becoming smaller and mass is increasing. On the left panel of the top row, where $B = 2.0 \times 10^{14}$g.cm^{-3}(left) the mass reaches maximum values (different for each hadronic equation of state but well bellow $1M_\odot$). After that, the mass is decreasing and starts to increase again. The stellar models when mass is decreasing with central pressure increasing are unstable against radial perturbations. After reaching minima the mass starts to increase, stellar models are stable again and the mass reaches new maxima. This second maximum is primarily given by the value of bag constant and is almost the same for all considered hadronic equations of state. We can see that for a small interval of masses the stable configuration may have two different radii. These objects are usually called twin stars - see Benić et al. (2015) for a more interesting case of high mass twin stars. In the case of our selection of equations of state, none of the hybrid star models meets the highest observed mass of $2.01 \pm 0.04M_\odot$. The only equation of state that meets the requirement is the UBS with $B = 4.5 \times 10^{14}$g.cm^{-3} but the mass is reached before the quark core starts to be present (see bottom right panel of Fig. 3 demonstrating there is no quark core before maximum mass is reached).

We calculated the moment of inertia of quark core $I_{\rm core}$ and we present its size relative to the total moment of inertia $I_{\rm tot}$ versus gravitational mass on Fig. 3. One can see that

[1] Haensel et al. (1986) were motivated by the investigation of strange stars, objects that are composed by a mixture of u,d and s quarks and the quark phase is energetically preferable up to zero pressure. In our case quark matter becomes energetically favorable if $P > P_{\rm pt}$.

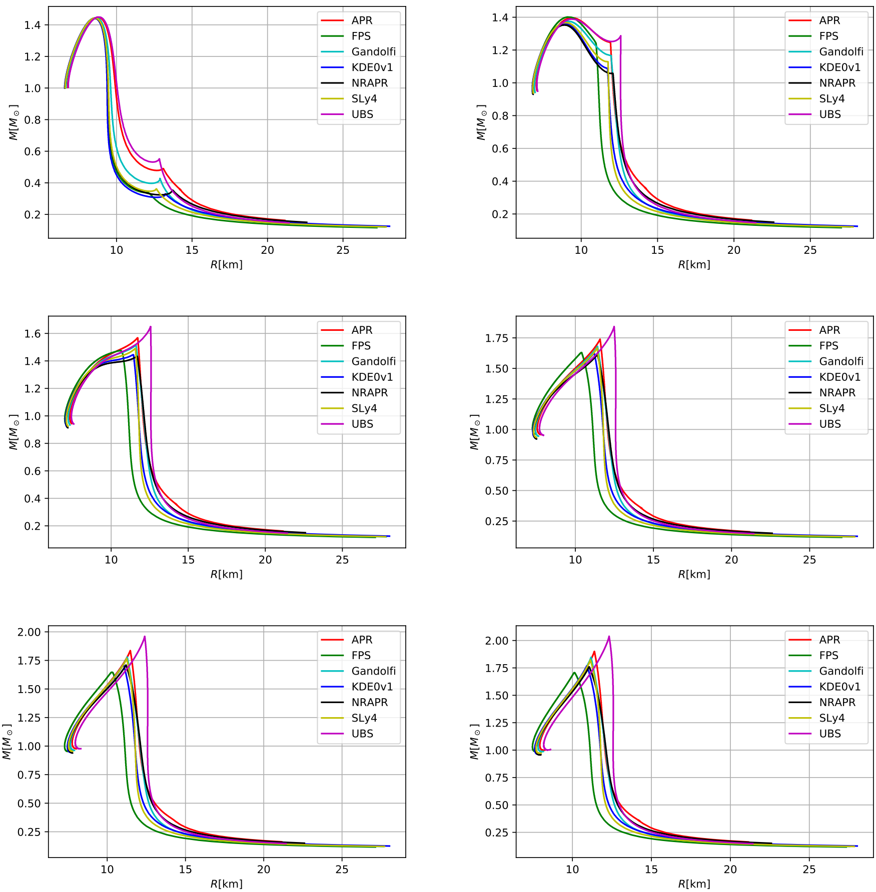

Figure 2. Mass vs radius of neutron stars with quark matter cores. Each panel represents stars having the same quark EoS but different hadronic equations of state. The values of Bag constants are (top left to bottom right) $B = \{2; 2.5; 3; 3.5; 4; 4.5\} \times 10^{14}\text{g.cm}^{-3}$ and all phase transitions are calculated using Maxwell construction. We can see that for a very small value of bag constant $B = 2 \times 10^{14}\text{g.cm}^{-3}$the maximum mass is dominated by the equation of state of quark matter, while in the case of higher values bag constant the hadronic equation of state plays an important role. Quark matter core is present in most of maximum mass configurations apart from the one with UBS EoS (see Fig 3).

the maximum size of quark core in stable compact stars corresponds to the lowest value of bag constant and that $I_{\text{core}}/I_{\text{tot}}$ is decreasing with increasing bag constant. Even for the highest values of B the quark core is present before reaching maximum mass except for UBS equation of state.

In this short proceeding, we presented a simple analysis of phase transition using several hadronic equations of state and combined each of them with the simplest form of MIT bag

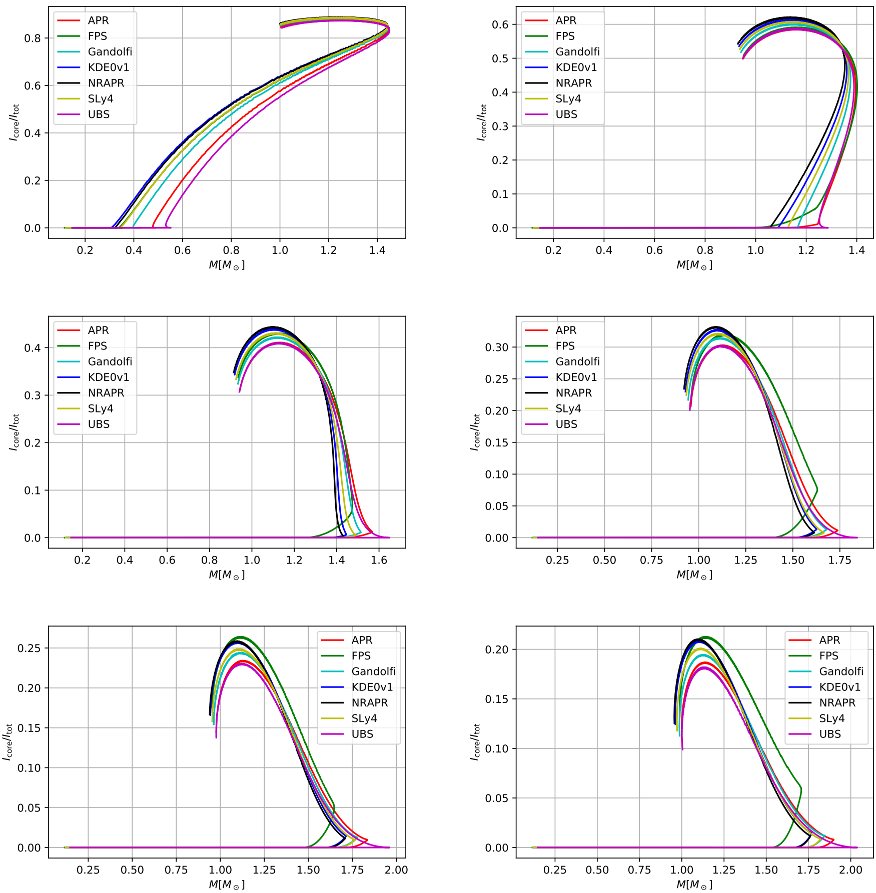

Figure 3. Moment of inertia of the quark core relative to the moment of inertia of the whole star. Each panel represents stars having the same quark EoS but different hadronic equations of state. The values of Bag constants are (top left to bottom right) $B = \{2; 2.5; 3; 3.5; 4; 4.5\} \times 10^{14}\mathrm{g.cm}^{-3}$ and all phase transitions are calculated using Maxwell construction. One can see that quark matter core starts to be present before maximum mass is reached in most of the cases, however, in the case of UBS the stable configurations can have quark core only in the two of investigated cases (top row).

model calculated for various values of bag constant. We showed that for low values of bag constant the maximum mass is determined by the value of bag constant while the hadronic equation of state plays an important role for higher values of bag constant. That is the starting point for our future investigation where we plan to use a more advanced equation of state of quark matter and investigate also rotating objects.

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