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Signal from neutron star obscured by oscillating accretion torus

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ABSTRACT
X-ray fluxes of low-mass neutron star binaries reveal rapid, nearly periodic changes corresponding to frequencies in the order of hundreds of hertz. Two closely related peaks often appear in the power spectral density and are designated as twin-peak Quasi-Periodic Oscillations (QPOs). Some QPO models attribute the observed effects to the torus oscillating in the inner region of the accretion flow. Since the observed variability is very strong, oscillations of a torus can be reflected in the observed light curves either by modulation of an accretion flow and/or by a periodic obscuration of a hot region on the neutron star surface. Applying a self-consistent description of the oscillations and full relativistic ray tracing, we analyse how the obscuration effect can affect the light curve detected by a distant observer. Within the same paradigm, we also investigate a possible product of torus instability and the implied signature of the Keplerian frequency in the light curve.

Keywords: Black hole – neutron star – X–ray binary – rapid variability

1 INTRODUCTION

Low-mass X-ray binaries (LMXBs) are among the most luminous sources in the X-ray universe. X-ray satellite observations on very short timescales showed that their luminosity varies periodically (or quasi-periodically). Some of the variability has already been explained in terms of the spin of the neutron star (NS), while a complex variability posing a broad phenomenology, the so-called quasi-periodic oscillations (QPOs), remains a mystery (e.g., Van der Klis, 2006; Török et al., 2022, and references therein).

The observed flux of LMXBs can change very rapidly, and the fastest variability is attributed to two distinguished frequency peaks in the power spectral density (PSD) of the signal in the range of hundreds of hertz, sometimes reaching kHz frequencies, provided that the binary contains a NS. In this case, in contrast to the black hole (BH) system observations, the two peaks are often clearly present simultaneously, which gave them the name

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twin-peak QPOs (see, e.g., van der Klis, 1998; McClintock and Remillard, 2006; Motta, 2016).

In our recent paper Török et al. (submitted) (hereafter Paper I), we contribute to solving the mystery of the rapid variability of LMXBs signal manifested by the twin-peak QPOs. We follow the path outlined by the series of previous works of other authors, who suggested that the changes in flux are caused by oscillations of an accretion torus in the innermost regions of the binary, very close to the central compact object (Abramowicz and Kluźniak, 2001; Kluźniak and Abramowicz, 2001; Rezzolla et al., 2003a,b; Abramowicz and Kluźniak, 2004; Bursa et al., 2004; Montero et al., 2004; Bursa, 2005; Horák, 2005; Török et al., 2005; Schnittman and Rezzolla, 2006; Abramowicz et al., 2007; Blaes et al., 2007; Ingram and Done, 2010; Mazur et al., 2016; Parthasarathy et al., 2017; de Avellar et al., 2018; Török et al., 2022, and references therein).

Oscillations of such torus can be reflected in the observed light curves either by the modulation of the accretion flow, giving rise to changes of the source luminosity due to a loss of energy of accreted matter reaching the NS, and/or by a periodic obscuration of the hot region on the central star's surface. The hot region corresponds to the boundary layer (BL) on the surface of a temperature much higher than that of the star. The accreted material, which reaches the star's surface there, slowly spreads towards the poles, and its temperature steadily drops towards the value on the surface. In principle, the amount of the accreted material, thus the temperature and local emissivity, may depend on the distance of the accretion torus to the star. For instance, when the torus is (radially) oscillating, the emissivity should follow those oscillations, resulting in periodic changes of the flux from the BL (Paczyński, 1987; Horák, 2005).

In this work, we follow our Paper I and consider two main scenarios. First, the modulation is caused by obscuring the NS surface with a radially and vertically oscillating torus. Second, the torus oscillates radially, but the other modulation is caused by the torus instability. In addition, we also compare these two scenarios to the consideration of radiating hot spots orbiting in the inner accretion region. Our present Proceedings paper is primarily intended as supplementary material for Paper I. We present here and discuss in more detail the setup of the simulations we have performed, and, furthermore, we introduce and analyse some results that did not fit the limited scope of Paper I.

2 MODELS, SIMULATIONS, AND THEIR SETUP

To validate the modulation model introduced in Paper I and to present its effects on the observable signal, we simulate the propagation of photons in a strong gravitational field. We use an analytical approximation for the inner accretion flow structure following the "Polish doughnut" model of Jaroszyński et al. (1980).

2.1 Models

We consider the following three scenarios. The justification for their selection and related references can be found in Paper I.

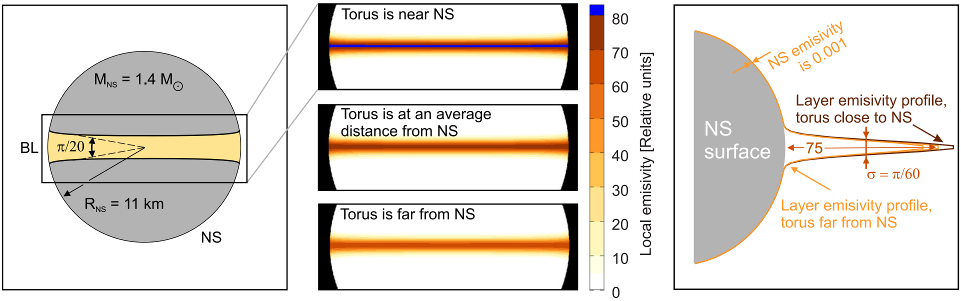

Figure 1. Setup of the BL simulations. The local emissivity of a NS and the hot BL on it. *Left:* Overall setup of our simulations. *Middle:* Maps showing the change in the BL local emissivity as the torus oscillates radially. The figure shows what a distant observer would see if relativistic changes in the photon energy were neglected (for the figure, we assume the g-factor $g = u^t_{\mathrm{observer}}/u^t_{\mathrm{emmitter}} = 1$). *Right:* Schematic profile of local emissivity of the star and BL system.

Model 1: Radially and vertically oscillating torus that periodically obscures the star's surface.

Model 2: Radially oscillating torus that is disrupted due to instabilities leading to the formation of a gap. The torus fragment orbits the NS, periodically uncovering part of its surface.

Model 3: A hot spot orbiting in the inner region of the equatorial accretion disk.

The first model considers a torus that oscillates radially and vertically and conceals the hot region of a BL on the NS's surface. This elaborate concept represents a continuation of ideas introduced in a series of papers by Abramowicz and Kluźniak (2001); Kluźniak and Abramowicz (2001); Abramowicz et al. (2007); Parthasarathy et al. (2017). The second model considers a fragmented torus as a product of instability. A gap forms within it, which orbits with the same (Keplerian) frequency as the original torus, periodically revealing the luminous BL on the surface (Török et al., 2016, 2022).

We expect that such configurations assumed within the first and second models can cause strong flux modulation with both radial and vertical (or orbital) frequencies. The third model considers a standard scenario of a hot spot orbiting within a thin accretion disk (Kluźniak et al., 1990; Karas and Bao, 1992; Stuchlík and Bao, 1992; Karas, 1999; Stella et al., 1999; Stella and Vietri, 1999). Since it is known that this model rather fails to explain the high amplitudes of QPOs, it is included merely for the sake of comparison.

To obtain light curves measured by a distant observer originating in a system with a compact object, one needs to utilise a code that calculates the propagation of light in a strong gravitational field (Cunningham and Bardeen, 1973; Karas et al., 1992; Beckwith and Done, 2005; Schnittman and Rezzolla, 2006; Dexter and Agol, 2009; Vincent et al., 2011; Chan et al., 2013; Bronzwaer et al., 2018; Prather et al., 2023). Within the context of the QPO modeling, ray-tracing was pioneered by Bursa et al. (2004); Schnittman and

Bertschinger (2004); Bursa (2005); Schnittman (2005); Schnittman et al. (2006b,a); Bakala et al. (2014, 2015); Mishra et al. (2017).

Here, we use the LSD relativistic ray-tracing code developed by our colleague Pavel (Bakala et al., 2015). In all examples in this contribution, except for those in Section 4, we assume the inclination of the observer, $i = 80°$, close to the equatorial plane.

2.2 Components of the system

The systems described by Models 1 – 3 consist of several components, which we describe in more detail:

Model 1: NS, BL on its surface, and an oscillating torus.

Model 2: NS, BL on its surface, and a product of torus instability.

Model 3: NS and a thin equatorial disk with a hot spot orbiting within it.

Each of the components has specific properties, which are summarised below.

NS: A spherical star with homogeneous local emissivity from the surface; in the relative units we use, the emissivity has a value of 0.001. Here, for simplicity, the NS does not rotate (see Figure 1).

BL: An equatorial region where the hot accreted material would fall on the star's surface. The local emissivity peaks at the value of 75 in relative units and decreases towards the poles to the value of the star's emissivity. Furthermore, it changes with the radial position of the nearby accretion torus centre, which reflects the accretion flow modulation by torus oscillations (see Figure 1). The four-velocity of the material within the BL at the equator corresponds to the Keplerian value of the specific angular momentum on the innermost stable circular orbit. As the material approaches the poles, it converges towards the four-velocity of the star's surface, with the same profile as the emissivity (Suleimanov and Poutanen, 2006; Gilfanov and Sunyaev, 2014). Thus, in our simplified case of a non-rotating star, the material at the pole is completely decelerated (see also the middle panel of Figure 4).

Torus: The torus configuration follows the setup developed by Bursa et al. (2004). The centre of the torus oscillates as

$$r = r_0 + \delta r \sin(\omega_{r,0}\, t)\,, \qquad z = 0 + \delta z \sin(\omega_{z,0}), \tag{1}$$

where δr and δz are the amplitudes in radial and vertical directions (in Model 2, $\delta z = 0$). Here, r_0 is the radial coordinate around which the torus oscillates and the position of the torus centre. Frequencies $\omega_{r,0}$ and $\omega_{z,0}$ correspond to the radial and vertical epicyclic frequencies of free test particles. The torus' local emissivity in our relative units is r_0/r (torus emissivity integrated over the whole surface from a distant observer is constant; see Figure 2). The four-velocity of the surface is given by the condition that the specific angular momentum is constant (see Figure 4). In Model 2, we consider a product of torus instability (e.g., Papaloizou and Pringle, 1984; Goldreich et al., 1986; Goodman

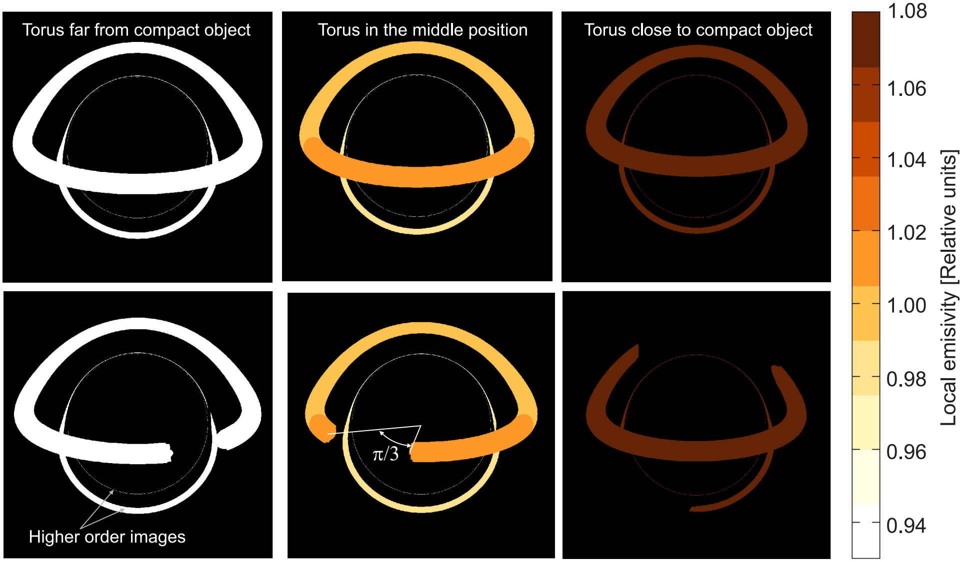

Figure 2. Local emissivity maps of the oscillating torus (without a NS star in the centre). The images show what a distant observer would see if relativistic changes in the photon energy were neglected. However, the relativistic effects on the photon trajectory are taken into account. Therefore, we see the characteristic relativistic shape of the torus with higher-order images. The local emissivity changes that occur with the radial oscillations of the torus are also visible. *Top*: Torus oscillating in the radial and vertical directions (Model 1). *Bottom*: Fragmented torus as a product of torus instability oscillating in the radial direction (Model 2).

et al., 1987), that causes fragmentation of the torus and formation of one or more gaps. We only model the case with one resulting fragment orbiting with the Keplerian frequency (see the bottom line of Figure 2).

Keplerian thin disk: An infinitesimally thin disk on Keplerian orbits in the equatorial plane, its local emissivity in relative units is proportional to $(r_{ms}/r)^3$, where $r_{ms} = 6.0M$ is the innermost stable circular orbit.

Hot spot: A spot with the emissivity of a Gaussian distribution (with $\sigma = 1/3M$) around its centre (see Figure 3). The spot oscillates in the radial direction. It has the same four-velocity as its centre.

3 RESULTS

In each case, we place the torus centre (or the spot) at the same radius, $r_0 = 6.75\,r_g$, which is frequently assumed for processes close to the ISCO. At this radius, the radial and vertical epicyclic frequencies are in a 3:1 frequency ratio (note that for a non-rotating NS,

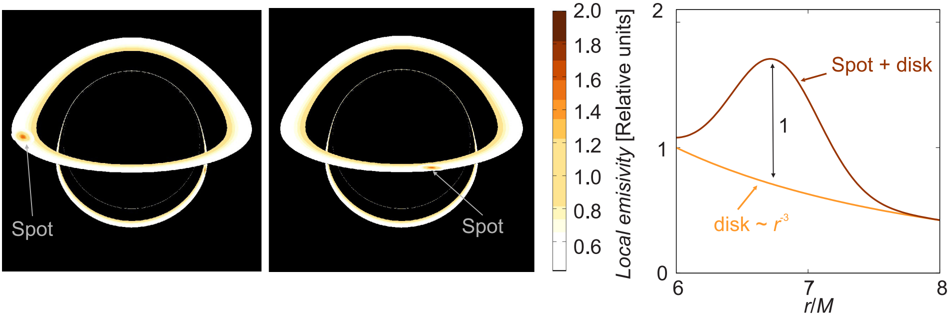

Figure 3. Local emissivity of the equatorial disk and a hot spot orbiting within the disk. *Left and Middle*: Maps of the torus emissivity at different moments during the orbital motion. The meaning is the same as in Figure 2. *Right:* Profile of the local emissivity of the spot and the equatorial disk in relative units.

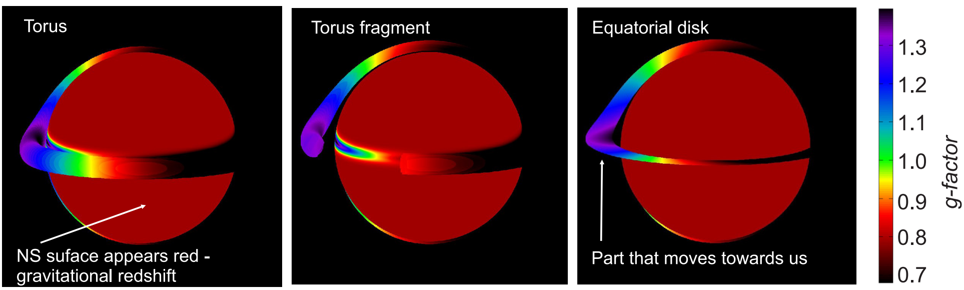

Figure 4. Maps of the g-factor illustrating the four velocities of the objects considered in our simulations. In the local system, the objects are green. The false colour scale corresponds to the human eye perception.

vertical epicyclic frequency equals the Keplerian frequency). This can have importance within the framework of QPO models considering resonances (Abramowicz et al., 2003). Nevertheless, for our simulations, the frequency ratio is not of primary importance.

For the above setup, we present the results of a thorough analysis of the images and light curves for a distant observer obtained from simulations of the previously described systems (four velocities of relevant objects are compared in Figure 4). In Figure 5, we show the synthetic light curves obtained for all models.

Apparently, it is possible to achieve significant amplitudes of variability of the measured flux using the configurations of Models 1 and 2. On the other hand, only small variability can be achieved using Model 3. The comparison of the intensity maps is illustrated in Figure 6.

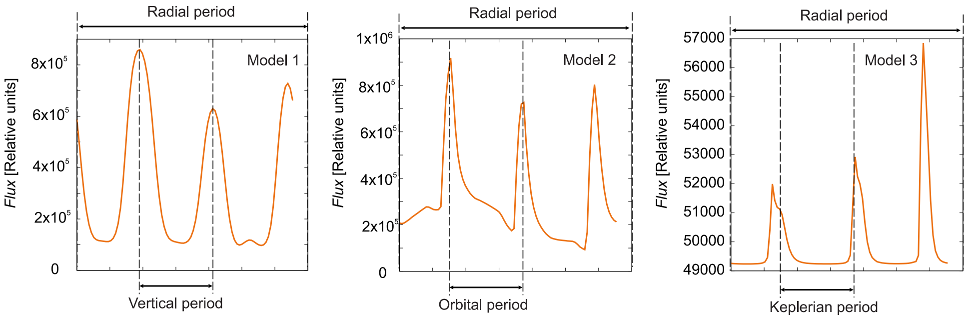

Figure 5. Light curves for all three models compared to each other, showing one period of the radial oscillations. *Left:* Model 1. *Middle:* Model 2. *Right:* Model 3. Note the different scales on the vertical axes - in the right panel, it does not start from 0. In our case, the vertical, orbital, and Keplerian periods have the same values but different physical meanings.

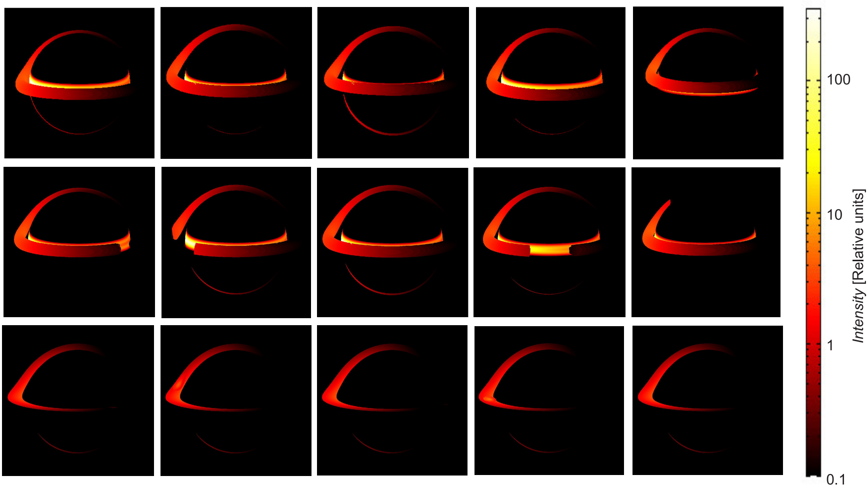

Figure 6. Intensity maps for all three models at different moments during the orbit. The configurations are the same as those corresponding to light curves presented in Figure 5. *Top*: Model 1. *Middle*: Model 2. *Bottom*: Model 3.

4 DISCUSSION AND CONCLUSIONS

This proceeding paper is intended as supplementary material to Paper I, extending the description of the models and presenting additional results that were beyond the scope of Paper I. We demonstrate how the torus, which periodically obscures the hot regions on the star, can modulate the observed light curves, leading to much higher variability amplitudes, usually absent in the other QPO models. The light curves of the two models (and the hot-spot model for comparison) are shown in Figure 5. Clearly, models incorporating the BL

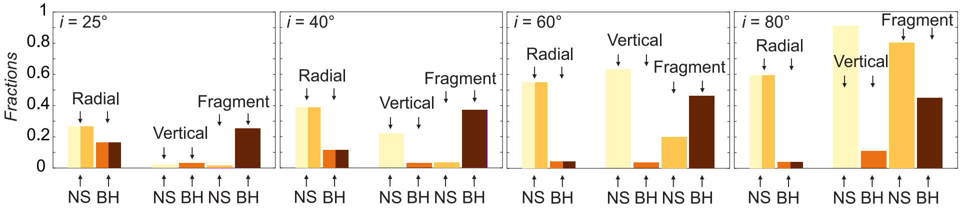

Figure 7. Fraction of fluxes affected by a given oscillatory (or orbital) motion for different observer's inclinations. Fractions correspond to the difference between the highest and lowest flux normalised to the highest flux value. The inclination angle increases from left to right.

obscuration into the modelling mechanism can potentially solve the problem of very high amplitudes of the observed QPOs.

To support our statement further, we quantified the fractions of flux affected by the individual oscillatory motions. For this purpose, we performed simulations for the radial and vertical oscillations separately. We also carried out an extra simulation for the non-oscillating but fragmented torus. The resulting fraction of fluxes affected by the given oscillatory motions for various observer's inclinations are compared in Figure 7. Furthermore, we compared the case of the luminous NS with a BL to that of a BH scenario where no radiation comes from the central object.

It is evident that a significant variability is induced in both Models 1 and 2 and almost all assumed configurations. Figure 7 also shows that the fragmented torus model (Model 2) can generate comparably high amplitudes of both QPOs also in the BH case, which the other models can reproduce only to a limited extent (see, e.g., Bursa et al., 2004).

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