On Reflection of Torus in the Kerr "Mirror"

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ABSTRACT
We investigate the effect of a reflective firewall surrounding the Kerr black hole. For a perfect fluid torus model, we construct a specific intensity map and corresponding specific intensity profile for impact parameter $\beta = 0$. We show that the "mirror" creates additional multi-ring structures in the image area below the photon orbit. We quantitatively show this effect in terms of the profile of the specific intensity $I_{\nu_0}(\alpha, \beta = 0)$ and show that the visibility of this multi-ring structure depends on the reflection efficiency of the mirror.

Keywords: Kerr black-hole – perfect fluid torus – raytracing – radiative transfer equation

1 INTRODUCTION

In the late '70s, Stephen Hawking's semi-classical treatment of black hole evaporation (Hawking, 1975) showed that black holes do radiate, which raised the still unresolved issue of so-called black hole information loss problem (Hawking, 1976). It exposes the conflict between quantum theory and general relativity. Gauge/gravity duality gives evidence that all information swallowed by a black hole is carried away by Hawking radiation. Now, it is believed that an external observer sees this information emitted by complex dynamical processes in close vicinity of the horizon, while the in-falling observer sees nothing special there. Almhein, Marolf, Polchinski, and Sully (AMPS) pointed out that the local quantum gravity, unitarity and "no drama" (assumption in-falling observer sees nothing special at the horizon) cannot be consistent with each other (Almheiri et al., 2013). They suggest giving up the "no drama" assumption, replacing it with the assumption that an in-falling observer should be terminated when hitting the so-called firewall. It is usually expected that firewalls lie on the black hole event horizons, however, in quantum mechanics, the boundaries are blurry, and the position of the horizon is uncertain up to fluctuations of the order of Planck length. In fact, a firewall may lie slightly inside the event horizons. In this case, it will fall

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down to physical singularity faster than the black hole size can shrink. However, it is supposed that a new firewall will be dynamically created on each fast-scrambling timescale. If the firewall lies inside of the horizon, it will be undetectable if one assumes that a firewall's location is determined by the past history of the Hawking evaporating black hole spacetime and is near where the event horizon would be if the evaporation rate were smooth, without quantum fluctuations. One can then show that quantum fluctuations of the evaporation rate in the future can migrate the event horizon to the inside of the firewall location and make the firewall naked and possibly detectable (Chen et al., 2016).

In this short contribution, we discuss the possible effect of reflecting mirror, whose existence is motivated by the firewall hypothesis, on the optical effects associated with the radiation emission of marginally stable torus orbiting central body generating Kerr spacetime with a horizon covered by the firewall (mirror). We start by introducing the mathematical formulation of the model. We present simulation results and make concluding remarks on calculated results.

2 THE MODEL

The gravitational field of a rotating compact object is modeled by Kerr spacetime defined by the dimensionless spin parameter $|a| \leq 1$. The mass of the black hole M is set to unity in what follows unless stated explicitly. In the Boyer-Lindquist coordinates (t, r, θ, ϕ) it takes the usual form (Kerr, 1963; Bardeen et al., 1972)

$$ds^2 = -\left(1 - \frac{2r}{\Sigma}\right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{A \sin^2\theta}{\Sigma} d\phi^2 - \frac{4a\, r \sin^2\theta}{\Sigma} dt d\phi, \tag{1}$$

where is $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta \equiv r^2 - 2r + a^2$, and $A = (r^2 + a^2)^2 - a^2\Delta \sin^2\theta$. The coordinate singularity, following from $\Delta = 0$, reveals the existence of the event horizon located at

$$r_h = 1 + \sqrt{1 - a^2}. \tag{2}$$

We envelope it with a reflective mirror that reflects infalling radiation with efficiency $0 \leq \eta \leq 1$, and we call it a "firewall". The firewall is located at the spacelike surface, orthogonal to $\partial/\partial r$ vector. The radius of that surface is

$$r_f \equiv r_h (1 + \epsilon), \tag{3}$$

where is the parameter $\epsilon > 0$.

The source of the radiation is perfect fluid, marginally stable torus. The pressure and density equipotential surface coincides with the potential (Kozłowski et al., 1978)

$$W(r, \theta) = \frac{1}{2} \log \left\{ \frac{\Delta [a^2 (1 + \cos 2\theta) + 2r^2]^2 \sin^2\theta}{4\Sigma \left[l_d^2 \left(\Delta - a^2 \sin^2\theta\right) - \sin^2\theta \,(A - 4a\, l_d\, r)\right]} \right\}, \tag{4}$$

where $l_d = const$ is the fluid angular momentum per unit mass. An element of the torus at given coordinates (r, θ) orbits the center with angular velocity (Kozłowski et al., 1978)

$$\Omega(r, \theta) = \frac{\left(2ar \sin^\theta + a^2 l_d \cos^2\theta\right) + l_d (r - 2)\, r}{\sin^2\theta \left[(A - a^2\Delta \cos^2\theta) \sin^2\theta - 2a l_d\, r\right]}. \tag{5}$$

Now, the outer surface of the torus is determined by $W(r,\theta) = W_0$. We assume that the specific emissivity profiles also coincide with equipotential surfaces of constant W, and we specify it as

$$j_\nu(r,\theta) \equiv j_0 \exp\left[-\frac{1}{\sigma^2}\left(\frac{W-W_c}{W_0-W_c}\right)\right] \text{ for } W < W_0 \text{ and 0 elsewhere.} \quad (6)$$

We introduced, here, the value of $W = W_c$ corresponding to the highest pressure/density being the local minimum of W.

The radiation follows null geodesics, identified by impact parameters l and q, in the Kerr spacetime. We divide the integration into two parts. First, there is a radial turning, we integrate the system of ordinary differential equations (Schee et al., 2023)

$$\frac{d^2u}{d\lambda^2} = \frac{1}{2\Sigma^2}\left(\frac{dU}{du} - 2\Sigma\frac{d\Sigma}{d\lambda}k^u\right), \quad (7)$$

$$\frac{d^2m}{d\lambda^2} = \frac{1}{2\Sigma^2}\left(\frac{dM}{dm} - 2\Sigma\frac{d\Sigma}{d\lambda}k^m\right), \quad (8)$$

$$\frac{dI_{\nu o}}{d\lambda} = g^2\, j_\nu. \quad (9)$$

Second, there is no turning point, a photon heads toward the collision with the "firewall", and we solve the set of differential equations (Schee et al., 2023)

$$\frac{d^2m}{du^2} = \frac{1}{2U^2}\left(\frac{dM}{dm} - U\frac{dU}{du}\frac{dm}{du}\right), \quad (10)$$

$$\frac{dI_{\nu o}}{du} = \frac{1}{k^u}g^2\, j_\nu. \quad (11)$$

Here we introduced effective potentials $U \equiv 1 + u^2(a^2 - l^2 - q + u(2((a-l)^2 + q) - a^2qu))$ and $M \equiv q + m^2(a^2 - l^2 - q - a^2m^2)$ and new coordinates $u \equiv 1/r$ and $m \equiv \cos(\theta)$ and $k^u = du/d\lambda$. The frequency shift, g, of photon emitted at particular fluid element of torus reads (Fanton et al., 1997)

$$g = \frac{\sqrt{1 - 2r(1 - a\Omega\sin^2\theta)^2/\Sigma - (r^2 + a^2)\Omega^2\sin^2\theta}}{1 - l\Omega}. \quad (12)$$

In the no-turning point case, the photon hits the firewall and is reflected. We reflect that photon in locally non-rotating frames (LNRF) in such a way that $k_r^{(0)} = k_i^{(0)}$, $k_r^{(1)} = -k_i^{(1)}$, $k_r^{(2)} = k_i^{(2)}$, and $k_r^{(3)} = k_i^{(3)}$; here the subscripts r and i represents the tetrad before and after reflection respectively. Now, the transformation of photon 4-momentum components from LNRF to coordinate frame reads $k^{(a)} = \Lambda_\mu^{(a)}k^\mu$, in particular $k^{(1)} = \Lambda_r^{(1)}\, k^r$ and therefore we obtain the reflection in terms of components relative to coordinate basis in the form $k_r^t = k_i^t$, $k_r^r = -k_i^r$, $k_r^\theta = k_i^\theta$, and $k_r^\phi = k_i^\phi$.

3 SIMULATION PARAMETERS AND RESULTS

We choose to illustrate the effect of the firewall on the specific intensity map of a marginally stable torus with the outer surface $W(r,\theta) = W_0 = -0.05$, angular momentum

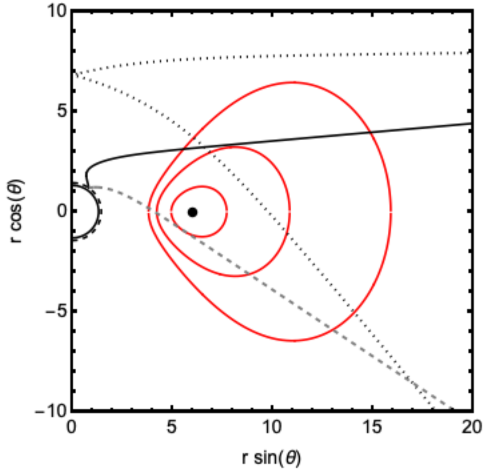

Figure 1. Meridional cross-section of the torus (red equipotential curves), firewall (black thick dashed line), ray with a turning point (black dotted line), and ray with no turning point (impacting segment dashed line and reflected segment black solid line).

$l_d = 3$ orbiting the Kerr black hole with spin parameter $a = 0.95$. The meridional cross-section of this configuration is in Fig. 1. The projection plane coordinates ranges are $(\alpha,\beta) = ([-20M, 20M], [-20M, 20M])$. Observer is located at $(u_o, \theta_o) = (10^{-4}M^{-1}, 15°)$, $(10^{-4}M^{-1}, 45°)$, and $(10^{-4}M^{-1}, 80°)$. For each point (α,β) we determine impact parameters (l, q) from formulas $l = \alpha\sqrt{1 - m_o^2}$ and $q = \beta^2 + m_o^2(\alpha^2 - a^2)$, and we integrate null geodesics using (7) -(9) in case there is a radial turning point or (10)-(11) in case there is no radial turning point. In the second case, the geodesics are parametrized with u, and we integrate them down to $u = 1/r_f$. If a firewall is present, the ray is reflected, and we integrate null geodesics outward to $u = 1/(1.5r_{\rm out})$ where $r_{\rm out}$ is the outermost radius of the torus.

The simulation results in $I_{\nu o}(\alpha,\beta)$ map presented in Fig. 2, where are nine density plots of torus image for three representative values of efficiency parameter $\eta = 0$ (left column), 0.4 (middle column), 1.0 (right column), and three representative values of observer inclination $\theta_o = 15°$ (top row), $45°$ (middle row) and $80°$ (bottom row). In each row, the observed specific intensity is normalised by the maximum of the observed specific intensity in case $\eta = 0$ (black hole). In this set of figures, we clearly see that the additional image structure in the region inside of the photon sphere is present, and its intensity relative to maximal intensity decreases with the observer inclination due to the strong enhancement of the Doppler effect becoming stronger for higher inclinations.

This effect is quantitatively illustrated in the $\beta = 0$ profiles of $I_{\nu o}(\alpha,\beta)$ plotted in Figs 3. In each figure there are three plots designated A (for $\eta = 0$), B (for $\eta = 0.4$), and C (for $\eta = 1.0$). The presence of the firewall (mirror) is revealed in the form of additional peaks in the region between the main peaks, which is also present in the black hole case. The amplitude is proportional to the reflection efficiency parameter η as expected.

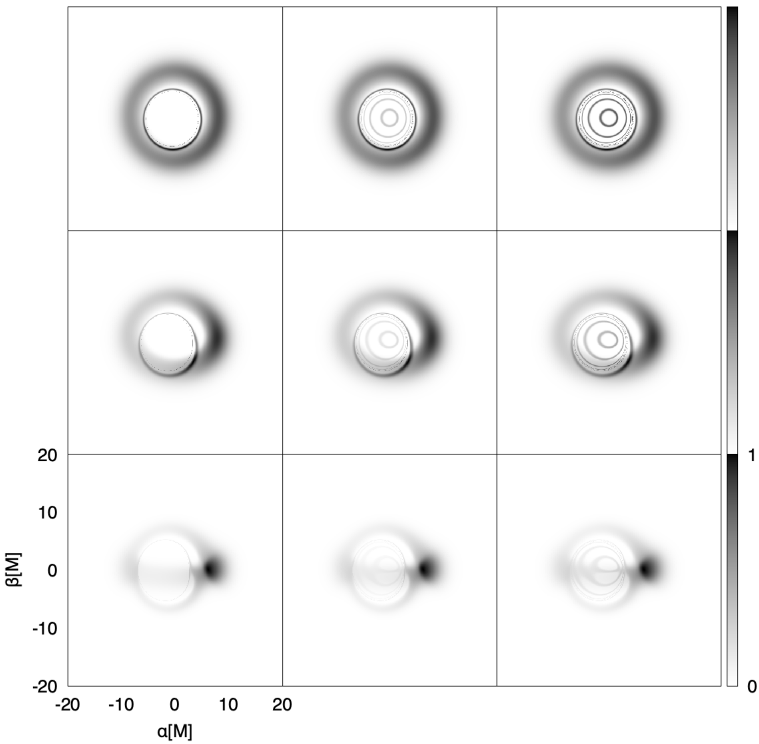

Figure 2. Illustrative examples of $I_{\nu 0}$ maps of disk image around a black hole (left) and firewall (right) with spin 0.95. The observer inclination is $\theta_o = 15°$ (top), $45°$ (middle), and $80°$ (bottom).

The intensity of reflected images does not depend on the firewall radius in the case of a stationary structure.

However, in the case of a system of temporal variability of emissivity, the deeper the radius of the firewall, the longer the time it takes a photon to reach the firewall, reflect, and climb out of the potential well. The effect of radiation echo takes place here, and the changes in the direct image of the disk intensity are observed before the changes in reflected images with a time delay corresponding to the depth of potential well (Hensh et al., 2022).

4 CONCLUSIONS

We presented a toy model of firewall a mirror with reflection efficiency η and radius $r_f = r_h(1+\epsilon)$ and simulated the specific intensity image maps and their α profiles, $I_{\nu o}(\alpha, \beta = 0)$. We showed that additional images of the torus appear in the region below the photon spherical orbit and that their intensity is proportional to the mirror reflection

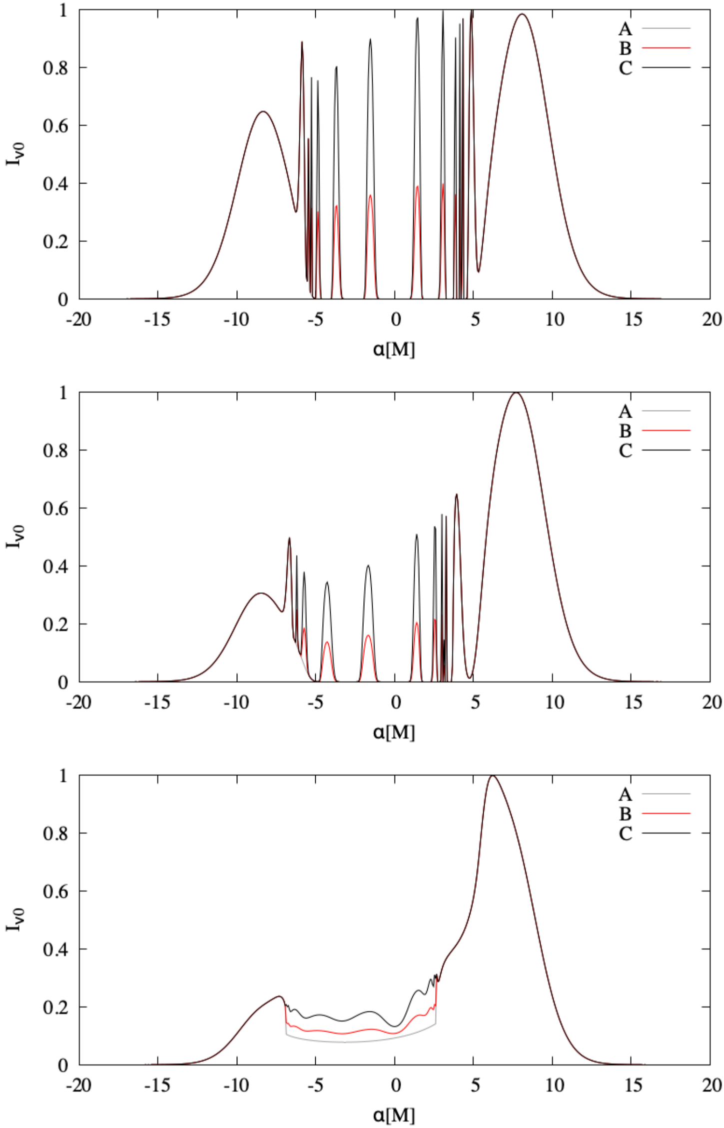

Figure 3. Profiles of $I_{\nu 0}(\alpha, \beta = 0)$ in case of spin parameter $a = 0.95$ and observer inclination $\theta_o = 15°, 45°$, and $80°$.

efficiency. We find that the observed specific intensity does not depend on the radius of the mirror. The visibility of the firewall depends strongly on the mirror reflection efficiency.

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