Quasinormal ringing of Bardeen spacetime

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ABSTRACT
We review recent calculations of quasinormal modes and asymptotic tails of the Bardeen spacetime interpreted as a quantum corrected Schwarzschild-like black holes. Massless electromagnetic and Dirac fields and massive scalar fields are considered. The first few overtones are much more sensitive to the change of the quantum correction parameter than the fundamental mode because such correction deforms the black hole geometry near the event horizon. While the asymptotic tails of massless fields are identical to those for the Schwarzschild case, the tails for a massive field differ from the Schwarzschild limit at both intermediate and asymptotic times.

Keywords: Regular spacetimes – quasinormal modes – outburst of overtones – quantum corrected black holes

1 INTRODUCTION

Quasinormal modes of black holes (Kokkotas and Schmidt, 1999; Nollert, 1999; Konoplya and Zhidenko, 2011) are a fundamental aspect of black hole physics and gravitational wave astronomy. These modes represent the characteristic oscillations and decay of perturbations around a black hole after an external perturbation, such as a merger or accretion event. These modes are characterized by complex frequencies, which have direct implications for the detection and interpretation of gravitational wave signals from black hole mergers by observatories like LIGO and Virgo (Abbott et al., 2016).

Quasinormal modes of the historically first model of the regular black holes given by the Bardeen spacetime (Bardeen, 1968) have been extensively studied in a great number of papers (see for instance Flachi and Lemos (2013); Toshmatov et al. (2015, 2019); Mahdavian Yekta et al. (2021); Rincón and Santos (2020); López and Ramírez (2022); Saleh et al. (2018) and reference therein). However, the Bardeen spacetime was considered there mainly as a solution of specific non-linear electrodynamics (Ayon-Beato and Garcia, 2000) which describes a black hole as a gigantic magnetic monopole with zero electric charge (Bronnikov, 2001).

Recently, the quasinormal spectrum of the Bardeen spacetime as a quantum corrected neutral black hole metric (Nicolini et al., 2019) has been considered in (Konoplya et al., 2023; Bolokhov, 2023b) with the emphasis to overtones behavior and asymptotic tails.

Here, we review these results and discuss three interesting phenomena related to the Bardeen black hole spectrum: a) outburst of overtones, b) arbitrarily long-lived modes of massive fields, and c) asymptotic tails.

2 BARDEEN SPACETIME AND THE WAVELIKE EQUATIONS

The spherically symmetric line element has the form

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{1}$$

where for the Bardeen spacetime, the metric function is

$$f(r) = 1 - \frac{2Mr^2}{\left(r^2 + l_0^2\right)^{3/2}}, \tag{2}$$

with M the Komar mass, and parameter l_0 is related to the ultraviolet cutoff (Nicolini et al., 2019). For $l_0 \neq 0$, the space-time in eq.(5) has horizons for $|l_0| \leq 4M/(3\sqrt{3})$.

The general relativistic equations for the scalar (Φ), electromagnetic (A_μ), and Dirac (Υ) fields in a curved spacetime can be written as follows:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) = 0, \tag{3a}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(F_{\rho\sigma}g^{\rho\nu}g^{\sigma\mu}\sqrt{-g}\right) = 0, \tag{3b}$$

$$\gamma^\alpha\left(\frac{\partial}{\partial x^\alpha} - \Gamma_\alpha\right)\Upsilon = 0. \tag{3c}$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor, γ^α are gamma matrices and Γ_α are spin connections in the tetrad formalism. Using separation of variables (for example, in case of the scalar field, employing the spherical symmetry and stationarity of the spacetime, we assume $\Phi \sim e^{-i\omega t}Y_\ell^m(\theta,\phi)\Psi(r)/r$, where Y_ℓ^m spherical harmonic function), after some algebra the above dynamical equations (3) take the wave-like form:

$$\frac{d^2\Psi}{dr_*^2} + (\omega^2 - V(r))\Psi = 0, \tag{4}$$

where the "tortoise coordinate" r_* is:

$$dr_* \equiv \frac{dr}{f(r)}. \tag{5}$$

The effective potentials for the scalar ($s = 0$) and electromagnetic ($s = 1$) fields can be written in a unified form:

$$V(r) = f(r)\frac{\ell(\ell+1)}{r^2} + (1-s)\frac{f(r)}{r}\frac{df(r)}{dr}, \tag{6}$$

where $\ell = s, s+1, s+2, \ldots$ are the multipole numbers. For the Dirac field ($s = 1/2$) the problem is reduced to two iso-spectral effective potentials

$$V_{\pm}(r) = W^2 \pm \frac{dW}{dr_*}, \quad W \equiv \left(\ell + \frac{1}{2}\right)\frac{\sqrt{f(r)}}{r}. \tag{7}$$

The iso-spectral wave functions can be transformed one into another by the Darboux transformation

$$\Psi_+ = q\left(W + \frac{d}{dr_*}\right)\Psi_-, \quad q = const, \tag{8}$$

so that it is sufficient to analyze the spectrum of only one of the potentials.

3 LONG LIVED QUASINORMAL MODES AND THE OUTBURST OF OVERTONES

The boundary conditions for quasinormal modes are purely outgoing wave at infinity and purely incoming wave at the event horizon so that

$$\Psi = \begin{cases} e^{i\omega r_*}, & \text{for } r_* \to +\infty \quad \text{(purely outgoing)}, \\ e^{-i\omega r_*}, & \text{for } r_* \to -\infty \quad \text{(purely ingoing)}. \end{cases} \tag{9}$$

In order to find low-lying quasinormal frequencies, the quick and relatively accurate method which was used in Konoplya et al. (2023); Bolokhov (2023b) is the 6th order WKB method (Konoplya, 2003; Konoplya et al., 2019a) with the Pade approximants (Matyjasek and Opala, 2017). The WKB method was effectively used in a great number of works (see, for example Kodama et al., 2010; Onozawa et al., 1996; Konoplya et al., 2019a, and references therein). In order to find accurate values of overtones with $n > \ell$, the convergent Leaver method was used (Leaver, 1985), while for the asymptotic tails, the time-domain integration (Gundlach et al., 1994) has been applied. The latter was used in various works as well (for instance, Konoplya and Fontana, 2008; Churilova and Stuchlik, 2020; Bolokhov, 2023a; Bronnikov and Konoplya, 2020) with a good concordance for the dominant frequencies. As all of these methods are broadly discussed in the literature, we will not discuss them here in detail.

Using the first order WKB approach and expanding in terms of $1/L$ and l_0, where $L = \ell + \frac{1}{2}$ we find the position of the maximum of the effective potential:

$$r_{\max} = 3M - \frac{5l_0^2}{6M} - \frac{65l_0^4}{216M^3} + O\left(l_0^6\right), \tag{10}$$

and the frequency

$$\begin{aligned}\omega = &\frac{L}{3\sqrt{3}M} - \frac{i(2n+1)}{6\sqrt{3}M} + l_0^2\left(\frac{L}{18\sqrt{3}M^3} + \frac{i(2n+1)}{54\sqrt{3}M^3}\right) \\ &+ l_0^4\left(\frac{17L}{648\sqrt{3}M^5} + \frac{7i(2n+1)}{324\sqrt{3}M^5}\right) + O\left(\frac{1}{L}, l_0^6\right).\end{aligned} \tag{11}$$

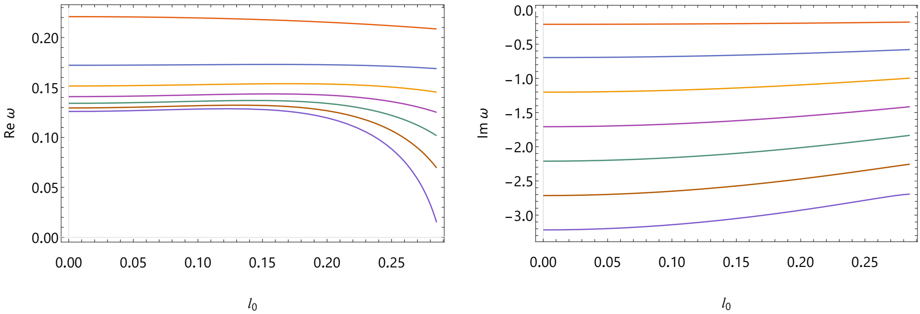

Figure 1. Quasinormal frequencies for the scalar perturbations ($\ell = 0$) and n from 0 to 6 (Konoplya et al., 2023).

Notice that in the eikonal limit, the WKB formula is exact, and the above formula satisfies the eikonal QNMs/null geodesics correspondence (Cardoso et al., 2009), though in general, the latter should be treated carefully since there are a number of exceptions from it (Konoplya and Stuchlík, 2017; Konoplya, 2023; Konoplya et al., 2019b).

From fig. 1, we notice that the overtones deviate from their Schwarzschild limits at an increasing with n rate, which reflects the fact that the Schwarzschild metric is deformed by the l_0 quantum correction mainly near the event horizon (Konoplya and Zhidenko, 2022).

When the scalar field has non-zero mass μ, the spectrum of the Schwarzschild and Reissner-Nordstrom black holes contains arbitrarily long-lived quasinormal modes at some values of μ (Ohashi and Sakagami, 2004). When μ is increased, the damping rate decreases, approaching zero as a kind of threshold at which the mode disappears from the spectrum, and the first overtone becomes the fundamental mode. This way, at particular values of the mass of the field, there exist the modes, called *quasi-resonances*, which are similar to standing waves. In Bolokhov (2023b), it was shown that this phenomenon also takes place for the massive scalar field in the Bardeen background and that the outburst of overtones takes place for such modes as well.

4 TELLING OSCILLATORY TAILS OF THE BARDEEN SPACETIME

At asymptotically late times, the massless scalar and gravitational fields for the Schwarzschild spacetime decay according to the following law (Price, 1972):

$$|\Psi| \sim t^{-(2\ell+3)}, \quad t \to \infty. \tag{12}$$

In fig. 2, we can see that the same law is fulfilled for the Bardeen spacetime.

When the massive term μ is turned on, the late-time behavior of the Reissner-Nordstrom black hole has two regimes (Koyama and Tomimatsu, 2002). At *asymptotic* times

$$\frac{t}{M} > (\mu M)^{-3}, \tag{13}$$

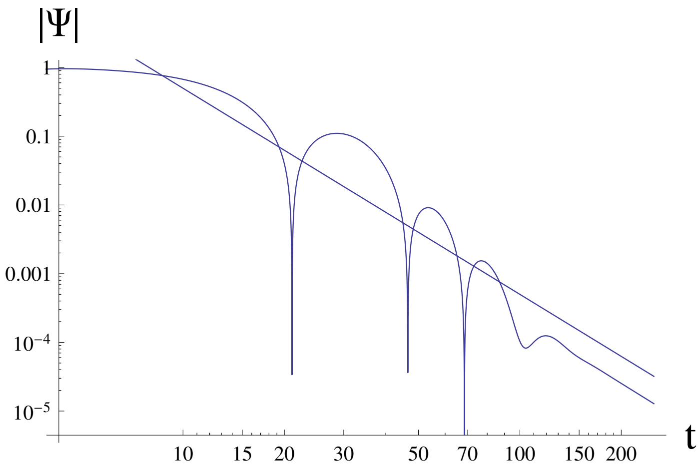

Figure 2. Time-domain profile according to (Konoplya et al., 2023) for scalar field perturbations $\ell = 0$ in the background of the quasi-extremal Bardeen black hole $l_0 = 0.707107$; $r_0 = 1$. Logarithmic plot with the line $\sim t^{-3}$.

the following decay law dominates

$$|\Psi| \sim t^{-5/6} \sin(\mu t), \quad t \to \infty. \tag{14}$$

For the Bardeen spacetime, as was shown in (Bolokhov, 2023b), the decay law is different:

$$|\Psi| \sim t^{-1} \sin(A(\mu)t), \quad t \to \infty, \tag{15}$$

where $A(\mu)$ is some function which could be approximately found by fitting the data for various values of μ. At the *intermediate late* times, corresponding to relatively small value of μM, the decay law for the Bardeen spacetime is (Bolokhov, 2023b),

$$|\Psi| \sim t^{-(\frac{8}{6}+\ell)} \sin(A(\mu)t), \tag{16}$$

which is also different from the Schwarzschild or Reissner-Nordstrom case (Koyama and Tomimatsu, 2002; Konoplya and Zhidenko, 2011).

5 CONCLUSIONS

We have reviewed recent studies (Konoplya et al., 2023; Bolokhov, 2023b) of quasinormal modes and evolution of perturbations of a test scalar, electromagnetic and Dirac fields in the vicinity of the Bardeen spacetime treated as a quantum corrected neutral black hole (Nicolini et al., 2019). The spectrum has a number of interesting and distinctive properties, such as outburst of overtones, long-lived quasinormal modes and different tail the behavior at asymptotic and intermediate times.

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