Stability of asymptotically flat (2+1)-dimensional black holes with Gauss-Bonnet corrections

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ABSTRACT
Using the integration of wave equation in the time-domain we show that scalar field perturbations around the (2 + 1)-dimensional asymptotically flat black hole with Gauss-Bonnet corrections are dynamically stable even for the near extreme values of the coupling constant.

Keywords: Scalar field perturbations – (2 + 1)-dimensional asymptotically flat black hole – Gauss-Bonnet corrections – quasinormal modes

1 INTRODUCTION

Black holes have been observed in the gravitational (Abbott et al., 2016) and electromagnetic (Akiyama et al., 2019; Goddi et al., 2016) spectra, while ongoing projects promise a great extension of the range of observations (Auclair et al., 2023). Quasinormal modes (Berti et al., 2009; Nollert, 1999; Kokkotas and Schmidt, 1999; Konoplya and Zhidenko, 2011) represent the characteristic oscillations that black holes undergo when perturbed, and they serve as a valuable tool for probing the black holes' fundamental properties. Our study explores the intricate interplay between black hole's quasinormal modes and their (in)stability. Usually, the strict mathematical proof of stability is a very difficult problem (see, for instance, Beyer, 2011). A clear evidence of the linear stability or instability is provided by the analysis of the quasinormal spectrum. If all the quasinormal modes are damped, the black hole is stable, while if there exists at least one unboundedly growing mode, it signifies the onset of instability. Therefore, the instability was extensively studied with the help of quasinormal modes spectra (Takahashi and Soda, 2010; Ishihara et al., 2008; Kodama et al., 2010; Dyatlov, 2011).

The particular black hole metric we will study here is the $(2+1)$-dimensional black hole obtained as a regularization (Glavan and Lin, 2020) of the Einstein-Gauss-Bonnet equations of motion in Konoplya and Zhidenko (2020) and further studied and generalized in Hennigar et al. (2020, 2021). Although the straightforward regularization in Glavan and Lin (2020) did not form a consistent theory in four dimensions, the black hole solution obtained within such a naive approach was also a solution of the well-defined theory formulated in Aoki et al. (2020).

Quasinormal modes of three-dimensional asymptotically AdS (BTZ) spacetimes were extensively studied (Cardoso and Lemos, 2001; Konoplya, 2004; Fontana, 2023), because of their importance in the AdS/CFT correspondence (Birmingham et al., 2002). However, no such analysis has been done so far for the $(2+1)$-dimensional asymptotically flat spacetimes, to the best of our knowledge. Here, we will show that the scalar field perturbations around the $(2+1)$-dimensional asymptotically flat black holes decay with time, even at the near-extreme values of the Gauss-Bonnet coupling.

2 BLACK HOLE METRICS AND WAVELIKE EQUATIONS

The metric of the (2+1)-dimensional black hole has the following form

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}x^2, \tag{1}$$

where the metric function is

$$f(r) = 1 - \frac{r^2}{2\alpha}\left(-1 + \sqrt{\frac{4\alpha\left(\Lambda\left(r^2 - r_{\mathrm{H}}^2\right) + \frac{\alpha}{r_{\mathrm{H}}^2} + 1\right)}{r^2} + 1}\right). \tag{2}$$

Here, r_{H} is the radius of the event horizon, Λ is the cosmological constant, α is the Gauss-Bonnet coupling constant. When $1 + 2\alpha/r_{\mathrm{H}}^2 > 0$ and the coupling constant $\alpha < 0$ at $\Lambda = 0$ the metric is asymptotically flat and perturbative in α. When the cosmological constant is negative, the metric is reduced to the BTZ one (Banados et al., 1992) up to the redefinition of constants. The general-covariant Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) = 0, \tag{3}$$

can be reduced to the wave-like form

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}r_*^2} + (\omega^2 - V(r))\Psi = 0, \tag{4}$$

where the "tortoise coordinate" r_* has the form:

$$\mathrm{d}r_* \equiv \frac{\mathrm{d}r}{f(r)}, \tag{5}$$

and the effective potential is

$$V(r) = f(r)\left(\frac{k^2}{r^2} + \frac{f'(r)}{2r} - \frac{f(r)}{4r^2}\right). \tag{6}$$

Here k is the multipole number.

3 EVOLUTION OF PERTURBATIONS

Quasinormal modes of asymptotically flat black holes are eigenvalues of the above wave-like equation satisfying particular boundary conditions: the waves must be purely outgoing at infinity and purely incoming at the event horizon. For analysis of the evolution of perturbations in time and consideration of the contribution of all the overtones so that the instability, if any, could be detected, we use the time-domain integration method (Gundlach et al., 1994), which was used in numerous works (for instance, Konoplya and Fontana, 2008; Churilova and Stuchlik, 2020; Bronnikov and Konoplya, 2020). It shows a good concordance with more accurate methods. The essence of this method is the integration of the wave equation in the framework of the null-cone coordinates $u = t - r_*$, $v = t + r_*$, using the following discretization (Gundlach et al., 1994),

$$\Psi(N) = \Psi(W) + \Psi(E) - \Psi(S) - \Delta^2 V(S) \frac{\Psi(W) + \Psi(E)}{4} + \mathcal{O}\left(\Delta^4\right). \tag{7}$$

Here, the points are: $N \equiv (u + \Delta, v + \Delta)$, $W \equiv (u + \Delta, v)$, $E \equiv (u, v + \Delta)$, $S \equiv (u, v)$, and the Gaussian impinging wave package are given on the null surfaces $u = u_0$ and $v = v_0$.

For checking the results obtained by the time-domain integration at $k > 0$ we used the 6th order WKB method with Padé approximants (Konoplya et al., 2019; Matyjasek and Opala, 2017; Konoplya, 2003).

Time-domain profiles show decaying profiles not only for small α (see Fig. 2) but also for the extreme α, as in Fig. 1. If the instability appeared, it would be governed by the non-oscillatory, i.e. purely imaginary, growing mode, as was proved in Konoplya et al. (2008) for generic static backgrounds.

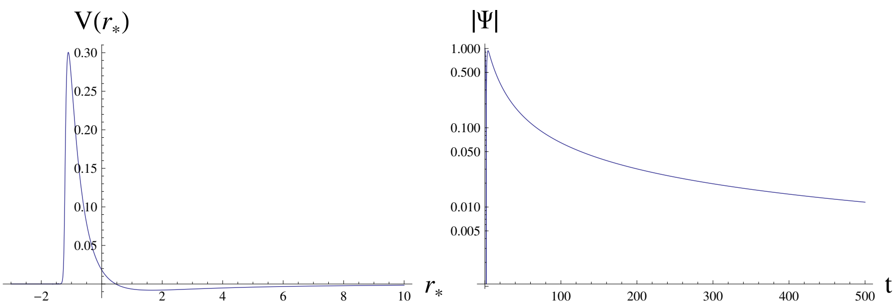

Figure 1. Left: Effective potential as a function of r_*. Right: Time-domain profile of perturbations. Here we have $k = 0$, $\Lambda = 0$, $\alpha = -0.49$. The Prony method suggests that the dominant modes are non-oscillatory $\omega = -0.001899i$

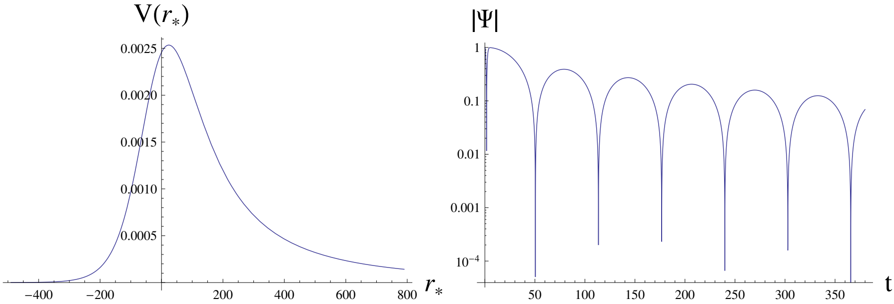

Figure 2. Left: Effective potential as a function of r_*. Right: Time-domain profile of perturbations. Here we have $k = 1$, $\Lambda = 0$, $\alpha = -0.01$. The Prony method gives $\omega = 0.050145 - 0.003598i$, while the 6th order WKB method with Pade approximants gives $\omega = 0.050155 - 0.003579i$

4 CONCLUSIONS

Here, we have shown that asymptotically flat (2 + 1)-dimensional black holes with Gauss-Bonnet corrections are stable even at the near-extreme values of the Gauss-Bonnet coupling. The full parametric range of stability and the other types of asymptotics could be studied in further research. Thus, allowing for a non-zero cosmological constant may bring instabilities to the system (Zhu et al., 2014; Konoplya and Zhidenko, 2014).

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