Harmonic oscillations of charged particles around weakly charged black hole

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ABSTRACT
The electric charge of a black hole is frequently neglected in many astrophysical scenarios due to unrealistically large values of the charge necessary for the Reissner-Nordström spacetime metric. Nevertheless, a variety of selective accretion processes could result in a black hole having a gravitationally weak electric charge. Thus, the motion of charged particles can be influenced by a combined effect of gravitational attraction and electrostatic interaction, which can be attractive or repulsive. This can lead to the existence of certain observational patterns in the non-thermal spectra of astrophysical black holes. In the present work, we study the harmonic oscillations of charged particles around a weakly charged black hole.

Keywords: Black hole – electric charge – harmonic oscillations

1 INTRODUCTION

The *no-hair theorem* states that the spacetime around black holes can be fully described by at most three metric parameters – black hole mass, spin, and electric charge. The charge of the black hole is usually neglected in astrophysical scenarios, justified by unrealistically large values of the charge required for its visible effect on the spacetime metric. Indeed, one can compare the gravitational radius of a black hole with the characteristic length of the charge $Q_{\rm G}$ of the Reissner-Nordström black hole, which gives the maximum charge value

$$\sqrt{\frac{Q_{\rm G}^2 G}{c^4}} = \frac{2GM}{c^2}, \quad \Rightarrow \; Q_{\rm G} = 2G^{1/2}M \approx 10^{31}\,\frac{M}{10M_{\odot}}\ {\rm Fr}. \tag{1}$$

This high value of charge is not yet justifiable in astrophysically relevant scenarios. On the other hand, there exist several astrophysical mechanisms based on selective accretion, in which the black hole can be weakly charged. The reader may refer for further details to

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Zajaček et al. (2018); Zajacek and Tursunov (2019); Tursunov et al. (2021). Wald mechanism (Wald, 1974) is a well-known method of charging black holes. It involves the natural induction of charge by the twisting of magnetic field lines caused by the black hole's rotation's frame-dragging effect. Therefore, the black hole and the surrounding magnetosphere should both accumulate a charge of $Q \sim 10^{18}$ Fr per solar mass. Recently, it was proposed that primordial black holes (PBHs) can be charged due to Poisson fluctuation at horizon crossing or high-energy particle collisions (Araya et al., 2023), and it was reported that the range of Q for PBHs is between 10^{-22} Fr and 10^{-6} Fr per kg, respectively.

In this paper, we study harmonic oscillations of charged particles in the vicinity of , a weakly charged black hole. Harmonic oscillations of charged particles in the vicinity of weakly magnetized black holes were previously studied in Kološ et al. (2015); Tursunov et al. (2016).

We use the metric signature $(- + + +)$, and the system of geometric units, in which $G = M = c = 1$.

2 DYNAMICS OF A CHARGED PARTICLE

2.1 Equations of motion

Spherically symmetric solution of Einstein's field equations, corresponding to the Schwarzschild spacetime metric, reads

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right), \tag{2}$$

where $f(r)$ is lapse function defined as

$$f(r) = 1 - \frac{2}{r}. \tag{3}$$

The only non-zero covariant component of the electromagnetic four-potential $A_\mu = (A_t, 0, 0, 0)$ is given by

$$A_t = -\frac{Q}{r}. \tag{4}$$

The anti-symmetric tensor of the electromagnetic field $F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$ has the only one independent nonzero component

$$F_{tr} = -F_{rt} = -\frac{Q}{r^2}. \tag{5}$$

Let us now consider the motion of a charged particle of mass m and charge q in the combined background of gravitational and electric fields. The motion of a charged particle is governed by the Lorentz equation in curved spacetime

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\alpha\beta}u^\alpha u^\beta - \frac{q}{m}F^\mu{}_\nu u^\nu = 0, \tag{6}$$

where u^μ is the four-velocity of the particle, τ is the proper time of the particle and $\Gamma^\mu_{\alpha\beta}$ – Christoffel symbols.

Two integrals of motion, which correspond to the temporal and spatial components of the classical four-momentum of the charged particle, can be introduced due to the symmetries of the background Schwarzschild metric as

$$\frac{P_t}{m} = -\mathcal{E} \equiv -\frac{E}{m} = u_t - \frac{qQ}{mr}, \tag{7}$$

$$\frac{P_\phi}{m} = \mathcal{L} \equiv \frac{L}{m} = u_\phi, \tag{8}$$

where $\mathcal{E}$ and $\mathcal{L}$ denote specific energy and specific angular momentum of the charged particle.

2.2 Effective potential

Using the normalization condition for the velocity of a massive particle $u^\mu u_\mu = -1$, one can derive the effective potential for the charged particle moving around a weakly charged Schwarzschild black hole in the form

$$V_{\text{eff}}(r) = \frac{\mathcal{Q}}{r} + \sqrt{f(r)\left(1 + \frac{\mathcal{L}^2}{r^2 \sin^2\theta}\right)}, \tag{9}$$

where $\mathcal{Q} = Qq/m$ is an electric interaction parameter.

The detailed analysis dynamics of a charged particle in the vicinity of the weakly charged black hole are represented in Tursunov et al. (2021); Pugliese et al. (2011).

3 HARMONIC OSCILLATIONS

One can observe an oscillatory motion of the test particle around its equilibrium position in the form of linear harmonic oscillations if the particle is displaced from a stable equilibrium position corresponding to a circular orbit at the position r_0, which is located at the minimum of the effective potential $V_{\text{eff}}(r, \theta)$ at $r = r_0$ and $\theta = \pi/2$. The following equations govern the particle's displacement in the case of harmonic oscillations around the stable orbit: $r = r_0 + \delta r$ and $\theta = \theta_0 + \delta\theta$

$$\ddot{\delta r} + \omega_r^2 \delta r = 0, \quad \ddot{\delta\theta} + \omega_\theta^2 \delta\theta = 0, \tag{10}$$

where dot denotes $\dot{x} = \mathrm{d}x/\mathrm{d}\tau$, here τ is the proper time of the particle. Then, the locally measured angular frequencies of the harmonic oscillations of the particle are given by

$$\omega_r^2 = \frac{\partial^2 V_{\text{eff}}}{\partial r^2}, \quad \omega_\theta^2 = \frac{1}{r^2 f(r)} \frac{\partial^2 V_{\text{eff}}}{\partial \theta^2}, \quad \omega_\phi = \frac{\mathrm{d}\phi}{\mathrm{d}\tau}. \tag{11}$$

Here, ω_ϕ is an axial (Keplerian) frequency, while ω_r and ω_θ are locally measured radial and latitudinal angular frequencies of the particle's harmonical oscillations, respectively. Applying the effective potential V_{eff} from (9), the locally measured latitudinal ω_θ, radial

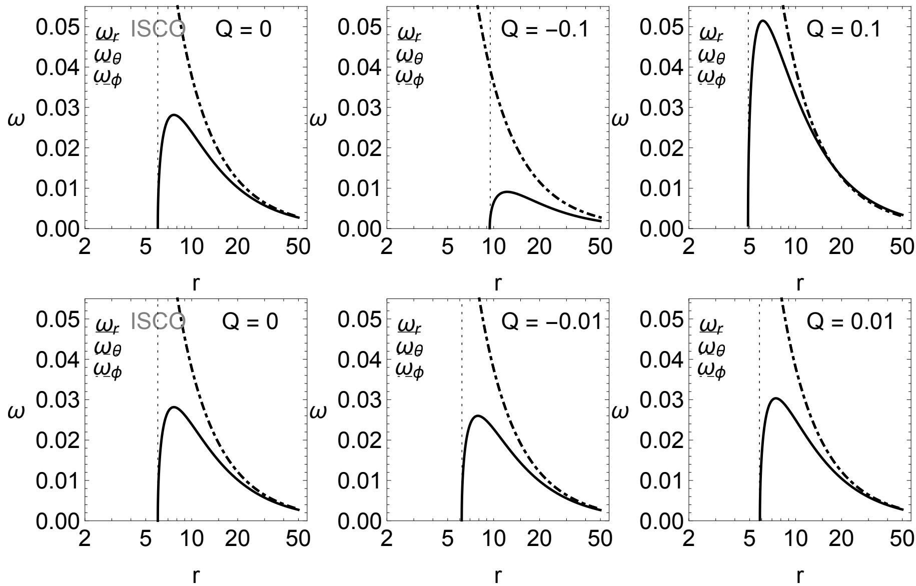

Figure 1. Dependence of locally measured frequencies ω_r, ω_θ, and ω_ϕ on radial distance from the black hole r. Due to the spherical symmetry of the fields, $\omega_\theta = \omega_\phi$. Here, we present three scenarios: the uncharged situation, where $Q = 0$ (left column); the attractive electrostatic interaction, where $Q < 0$ (middle column); and the repulsive electrostatic interaction, where $Q > 0$ (right column).

ω_r, and axial (Keplerian) ω_ϕ frequencies of the charged particle harmonic oscillations in the field of a weakly electrically charged black hole take the explicit form

$$\omega_\theta^2 = \frac{\mathcal{L}^2}{r^4}, \tag{12}$$

$$\omega_r^2 = \frac{3\mathcal{L}^2 r^2 f^3 - 2r^3 \mathcal{E}^2\left(\mathcal{E}^2 - Q\right) - Q^2\left(4 + 3fr^2\right)}{f^2 r^6}, \tag{13}$$

$$\omega_\phi = \frac{d\phi}{d\tau} = U^\phi = \frac{\mathcal{L}}{g_{\phi\phi}} \equiv \frac{\mathcal{L}}{r^2 \sin^2\theta}, \tag{14}$$

where $f = 1 - 2/r$ is the lapse function, $\mathcal{L}$ is the spefic angular momentum and $\mathcal{E}$ is the specific energy, which is $\mathcal{E} = V_{\rm eff}(r)$. Figure 1 illustrates the dependence of the locally measured frequencies $\omega_\theta(r)$, $\omega_r(r)$, and $\omega_\phi(r)$ on the radial coordinate r for the values of the electric interaction parameter $Q = 0, \pm 0.01, \pm 0.1$. Locally measured harmonic oscillations are expressed in physical units. By factor c^3/GM, a dimensionless form can be obtained.

4 CONCLUSION

This work presents the study of harmonic oscillations of charged particles in the vicinity of a weakly charged Schwarzschild black hole. We have derived equations for the angular frequencies of the harmonic oscillations of the particle as measured by a local observer. Additionally, we have constructed the radial dependence on the locally measured angular frequencies for various values of the electric interaction parameter. The obtained results can further be applied in relation to the quasiperiodic oscillations (QPOs) phenomena observed in black hole microquasars (Kološ et al., 2017; Tursunov and Kološ, 2018),

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