Quasinormal spectrum in the asymptotically safe gravity

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ABSTRACT
Asymptotically safe gravity is based on the idea of the dependence of the gravitational coupling upon the distance from the origin, approaching its classical value in the weak field regime. We consider three cases of identifying the cut-off parameter in the asymptotically safe gravity, leading to the three distinctive models for black holes. We find that the deviation of the fundamental mode from the Schwarzschild limit is a few per cent, in contrast to the higher overtones, where the deviation reaches hundreds of per cent, even when the fundamental mode almost coincides with the Schwarzschild mode. This behavior is connected with the fact that the quantum correction to the black hole spacetime is strong near the event horizon but quickly falls off with distance and is negligible near the peak of the effective potential surrounding the black hole.

Keywords: Regular spacetimes – quasinormal modes – outburst of overtones – asymptotically safe gravity

1 INTRODUCTION

The investigation of quasinormal modes (QNMs, Kokkotas and Schmidt, 1999; Nollert, 1999; Konoplya and Zhidenko, 2011) of black holes has become crucial in comprehending how black holes respond to perturbations. QNMs, representing the characteristic damped oscillations of black holes, encapsulate vital information about their fundamental properties. Their observation is facilitated by gravitational interferometer systems like LIGO and Virgo (Abbott et al., 2016). Simultaneously, the concept of asymptotically safe gravity (Bonanno and Reuter, 2000) has emerged as a theoretical framework aimed at addressing issues regarding the renormalization of gravity in the quantum domain.

This report explores the relationship between QNMs of black holes and the hypothesis of asymptotically safe gravity. We conduct an extensive examination of quasinormal modes of test fields for three black-hole models within asymptotically safe gravity, demonstrating that the overtones are considerably more sensitive to quantum corrections than the fundamental mode. This phenomenon, termed “the outburst of overtones”, is associated with the

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distortion of black hole geometry near its event horizon (Konoplya and Zhidenko, 2022). This report provides an overview of the primary findings concerning quasinormal spectra of black holes in asymptotically safe gravity. Further details can be explored in Konoplya et al. (2022, 2023).

2 BLACK HOLE METRICS AND WAVELIKE EQUATIONS

The metric of a spherically-symmetric black hole has the following form

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega^2. \tag{1}$$

In asymptotically safe gravity, Newton's coupling now depends on r. The basic constituent of the theory is the cut-off parameter. There are three known ways to identify the cut-off parameter of the theory:

(1) Identification of the cut-off parameter as a modified proper distance leads to the *Bonanno-Reuter metric* (Bonanno and Reuter, 2000). The metric function for the Bonanno-Reuter spacetime is

$$f(r) = 1 - \frac{2Mr^2}{r^3 + \frac{118}{15\pi}\left(r + \frac{9}{2}M\right)}, \tag{2}$$

where M is the black hole mass measured in units of the Planck mass.

(2) Identification of the cut-off parameter as a function of the Kretschmann scalar (Held et al., 2019) leads to the metric coinciding with the *Hayward metric* (Hayward, 2006):

$$f(r) = 1 - \frac{2r^2/M^2}{r^3/M^3 + \gamma}. \tag{3}$$

Here, the event horizon exists whenever $\gamma \leq 32/27$.

(3) Starting from a classical Schwarzschild solution, the backreaction effects produced by running Newton's coupling are taken into account iteratively. This way, a kind of coordinate independent iterative procedure for identification suggested in Platania (2019) leads to the Dymnikova black hole (Dymnikova, 1992)

$$f(r) = 1 - \frac{2M}{r}\left(1 - e^{-\dfrac{r^3}{2l_{\rm cr}^2 M}}\right). \tag{4}$$

Here, $l_{\rm cr}$ is a critical length scale below which the modifications owing to the running of Newton's constant become negligible. The maximal value of $l_{\rm cr}$ at which the event horizon still exists is

$$l_{\rm cr} \approx 1.138\,M,$$

where M is the mass measured in units of length.

The general relativistic equations for the scalar (Φ), electromagnetic (A_μ), and Dirac (Υ) fields in a curved spacetime can be cast to the wave-like form:

$$\frac{d^2\Psi}{dr_*^2} + \left(\omega^2 - V(r)\right)\Psi = 0, \tag{5}$$

where the "tortoise coordinate" r_* is:

$$dr_* \equiv \frac{dr}{f(r)}. \tag{6}$$

The effective potentials for the scalar ($s = 0$) and electromagnetic ($s = 1$) fields can be written in a unified form:

$$V(r) = f(r)\frac{\ell(\ell+1)}{r^2} + (1-s)\cdot\frac{f(r)}{r}\frac{df(r)}{dr}, \tag{7}$$

where $\ell = s, s+1, s+2, \ldots$ are the multipole numbers. For the Dirac field ($s = 1/2$), the problem is reduced to two iso-spectral effective potentials

$$V_\pm(r) = W^2 \pm \frac{dW}{dr_*}, \quad W \equiv \left(\ell + \frac{1}{2}\right)\frac{\sqrt{f(r)}}{r}. \tag{8}$$

The isospectral wave functions $\Psi_\pm$ can be transformed one into another by using the Darboux transformation:

$$\Psi_+ = q\left(W + \frac{d}{dr_*}\right)\Psi_-, \quad q = const. \tag{9}$$

Therefore, it is sufficient to study only one of the iso-spectral cases.

3 QUASINORMAL MODES

Quasinormal modes represent the appropriate oscillation frequencies that govern the evolution of perturbations during the intermediate to late stages, known as the ringdown phase. These modes correspond to solutions of the master wave equation (5) subject to the following boundary conditions for the wave function $\Psi \propto e^{i\omega(r_*-t)}$:

$$\begin{aligned} \Psi &\sim \textit{pure outgoing wave}, \quad r_* \to +\infty, \\ \Psi &\sim \textit{pure ingoing wave}, \quad r_* \to -\infty. \end{aligned} \tag{10}$$

For finding dominant quasinormal modes, we used the 6th order WKB method (Konoplya, 2003; Konoplya et al., 2019) with the Pade approximants (Matyjasek and Opala, 2017). The WKB method has been broadly applied for finding quasinormal modes of black holes and wormholes (for example in Kodama et al., 2010; Onozawa et al., 1996; Konoplya et al., 2019; Bolokhov, 2023). In order to find precise values of modes with arbitrary relation between the multipole number and overtone, the Leaver method is used (Leaver, 1985), which is based on the convergent procedure. Another convergent technique we used is the Bernstein polynomial method (Konoplya and Zhidenko, 2023), though only several

n	ω (Bonanno-Reuter)	ω (Schwarzschild)
0	$0.066877 - 0.020549i$	$0.062066 - 0.023122i$
1	$0.059764 - 0.063499i$	$0.053629 - 0.073417i$
2	$0.047392 - 0.110619i$	$0.043693 - 0.131297i$
3	$0.030448 - 0.166115i$	$0.036544 - 0.192977i$
4	$0.019951 - 0.224263i$	$0.031639 - 0.255638i$
5	$0.012209 - 0.290487i$	$0.028063 - 0.318481i$
6	$0.000983 - 0.354117i$	$0.025304 - 0.381317i$
7	$0.000 - 0.420i$	$0.023081 - 0.444100i$

Table 1. Dominant quasinormal modes for the electromagnetic perturbations ($\ell = 1$) of the Bonanno-Reuter black hole ($M = 4$) and the corresponding modes for the Schwarzschild black hole, according to Konoplya et al. (2022).

first overtones can be found during reasonable computing time. Finally, in order to see the evolution of perturbations in time, the time-domain integration developed in Gundlach et al. (1994) has been applied. This method was also used in a great number of publications (see, for example, Konoplya and Fontana, 2008; Churilova and Stuchlik, 2020; Bronnikov and Konoplya, 2020 and references therein) with a very good concordance with other methods for the fundamental mode.

3.1 Bonanno-Reuter metric

Using the first-order WKB formula, we can obtain quasinormal modes in the high multipole number $\ell \to \infty$ regime in analytic form. For this, we will use the expression for the position of the peak of the effective potential, which for the Bonanno-Reuter metric is located at:

$$r_{\max} \approx 3M - \frac{1652}{135\pi M} - \frac{5071817}{54675\pi^2 M^3} - \frac{28261793432}{22143375\pi^3 M^5}. \tag{11}$$

Then, the WKB formula yields

$$Im(\omega) = \frac{\left(n + \frac{1}{2}\right)}{3\sqrt{3}M}\left(1 - \frac{1298}{405\pi M^2} + O\left(\frac{1}{M^4}\right)\right), \tag{12}$$

$$Re(\omega) = \frac{\left(\ell + \frac{1}{2}\right)}{3\sqrt{3}M}\left(1 + \frac{59}{27\pi M^2} + O\left(\frac{1}{M^4}\right)\right). \tag{13}$$

Observing table 1, it becomes evident that as n increases, the deviation of the overtones amplifies, while the fundamental mode only exhibits a slight deviation from the Schwarzschild limit. Additionally, at $n = 7$, a purely imaginary (non-oscillatory) mode emerges in the spectrum, which is not an algebraically special one.

n	ωM (Hayward)	ωM (Schwarzschild)
0	$0.113494 - 0.089160i$	$0.110455 - 0.104896i$
1	$0.066731 - 0.319873i$	$0.086117 - 0.348053i$
2	$0.041068 - 0.576924i$	$0.075742 - 0.601079i$
3	$0.021679 - 0.833067i$	$0.070410 - 0.853678i$
4	$0.000000 - 1.082236i$	$0.067074 - 1.105630i$
5	$0.001449 - 1.317232i$	$0.064742 - 1.357140i$

Table 2. Quasinormal modes for the $\ell = 0$ scalar perturbations of the Hayward black hole ($\gamma = 1$) and the corresponding modes for the Schwarzschild black hole, according to Konoplya et al. (2022).

3.2 Hayward metric

In a similar fashion, we obtain the following expressions for the Hayward metric for the location of the peak of the effective potential,

$$r_{\max} \approx 3M - \frac{2\gamma M}{9} - \frac{\gamma^2 M}{27} - \frac{70\gamma^3 M}{6561} - \frac{665\gamma^4 M}{177147}, \tag{14}$$

and the eikonal quasinormal frequencies,

$$Im\,(\omega) = \frac{\left(n+\frac{1}{2}\right)}{3\sqrt{3}M}\left(1 - \frac{2}{27}\gamma + O\left(\gamma^2\right)\right), \tag{15}$$

$$Re\,(\omega) = \frac{\left(\ell+\frac{1}{2}\right)}{3\sqrt{3}M}\left(1 - \frac{2}{54}\gamma + O\left(\gamma^2\right)\right). \tag{16}$$

According to observations in Cardoso et al. (2009), there exists a correspondence between the eikonal quasinormal modes and characteristics of null geodesics: The real and imaginary parts of ω are multiples of the frequency and instability timescale of the circular null geodesics, respectively. While we confirm this observation for the specific case considered in asymptotically safe gravity, generally, this correspondence does not hold for modes with $\ell \gg n$ that cannot be accurately reproduced by the WKB formula (Konoplya and Stuchlík, 2017; Konoplya, 2023).

Table 2 shows that there is an outburst of overtones for the Hayward metric as well.

3.3 Dymnikova black hole

As can be seen from Table 3, the overtones deviate at a stronger rate from their Schwarzschild values when n is increased. For all three types of metrics, the deformation of the geometry occurs mainly near the event horizon, while in the far zone, the metrics merge with the Schwarzschild one. The overtones are highly sensitive to these near-horizon deformations.

l_{cr}	$n = 0$	$n = 1$	$n = 2$	$n = 3$
0	$0.24826 - 0.09249i$	$0.2145 - 0.2937i$	$0.175 - 0.525i$	$0.146 - 0.772i$
0.7	$0.24823 - 0.09247i$	$0.2141 - 0.2935i$	$0.173 - 0.525i$	$0.140 - 0.772i$
0.75	$0.24814 - 0.09239i$	$0.2135 - 0.2930i$	$0.170 - 0.523i$	$0.135 - 0.768i$
0.8	$0.24804 - 0.09226i$	$0.2125 - 0.2922i$	$0.166 - 0.521i$	$0.123 - 0.765i$
0.85	$0.24790 - 0.09200i$	$0.2112 - 0.2909i$	$0.160 - 0.518i$	$0.103 - 0.762i$
0.9	$0.24766 - 0.09159i$	$0.2088 - 0.2886i$	$0.149 - 0.512i$	$0.06 - 0.74i$
0.95	$0.2473 - 0.09085i$	$0.206 - 0.2849i$	$0.13 - 0.50i$	$0.06 - 0.81i$
1.0	$0.2468 - 0.08986i$	$0.201 - 0.280i$	$0.107 - 0.508i$	$0.05 - 0.8i$
1.05	$0.24614 - 0.08855i$	$0.195 - 0.276i$	$0.107 - 0.514i$	$0.04 - 0.8i$
1.1	$0.24516 - 0.08699i$	$0.1892 - 0.2731i$	$0.091 - 0.519i$	$0.05 - 0.87i$

Table 3. Quasinormal modes found by the Leaver method for $\ell = 1$, electromagnetic perturbations; $M = 1$. The metric is approximated by the 17th-order parametrization in Konoplya et al. (2023). The Schwarzschild limit corresponds to $l_{cr} = 0$.

4 CONCLUSIONS

We have reviewed recent results obtained in Konoplya et al. (2023, 2022) regarding the behavior of overtones in various black hole models within asymptotically safe gravity. Despite different approaches to identifying the cutoff parameters, a qualitatively similar feature is observed in all three cases: an outburst of overtones that convey information about the geometry of the event horizon.

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